Appendices for: The Shale Revolution and the Dynamics of the Oil Market*

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A Derivation of the structural dynamic model

Conventional and shale fringe producers ($j = f, s$ respectively) take prices as given. They choose current utilisation ($u_{j,t}$ and next period capacity ($k_{j,t}$) to maximise the present value of profits. The first order conditions for the fringe producers are given by:

$$\frac{\partial \pi_{j,t}}{\partial u_{j,t}} = 0 \tag{A.1}$$

$$\frac{\partial \pi_{j,t}}{\partial k_{j,t}} + \underset{t}{E}\left[\beta \frac{\partial \pi_{j,t+1}}{\partial k_{j,t}}\right] = 0 \tag{A.2}$$

where $\pi_{j,t}$ is period t profits defined by equation (5). Equation(A.2) describes the intertemporal trade-off between the costs of changing capacity ($\frac{\partial \pi_{j,t}}{\partial k_{j,t}}$) and future revenue ($\frac{\partial \pi_{j,t+1}}{\partial k_{j,t}}$). The dominant producer in time period t can be described as:

$$\begin{aligned}
&\max_{u_{o,t},\ k_{o,t},\ P_t,\ u_{j,t},\ k_{j,t},\ j=f,s} \quad \underset{t}{E}\sum_{i=0}^{\infty}\beta^i \pi_{o,t+i} \\
&+ \lambda^p_{t+i}\left(P\left(u_{o,t+i}k_{o,t-1+i} + u_{f,t+i}k_{f,t-1+i} + u_{s,t+i}k_{s,t-1+i},\right.\right. \\
&\left.\left. u_{o,t-1+i}k_{o,t-2+i} + u_{f,t-1+i}k_{f,t-2+i} + u_{s,t-1+i}k_{s,t-2+i}, x_{d,t+i}\right) - P_{t+i}\right) \\
&+ \lambda^u_{f,t+i}\left(-\frac{\partial \pi_{f,t+i}}{\partial u_{f,t+i}}\right) \\
&+ \lambda^k_{f,t+i}\left(-\left[\frac{\partial \pi_{f,t+i}}{\partial k_{f,t+i}} + \beta\frac{\partial \pi_{f,t+i+1}}{\partial k_{f,t+i}}\right]\right) \\
&+ \lambda^u_{s,t+i}\left(-\frac{\partial \pi_{s,t+i}}{\partial u_{s,t+i}}\right) \\
&+ \lambda^k_{s,t+i}\left(-\left[\frac{\partial \pi_{s,t+i}}{\partial k_{s,t+i}} + \beta\frac{\partial \pi_{s,t+i+1}}{\partial k_{s,t+i}}\right]\right)
\end{aligned}$$

The terms $\lambda^p_{t+i}, \lambda^u_{f,t+i}, \lambda^k_{f,t+i}, \lambda^u_{s,t+i}, \lambda^k_{s,t+i}$ are Lagrange multipliers on the market clearing constraint, and the first order conditions of fringe production decisions ((A.1) and (A.2)). The Lagrange multiplier λ^p_t reflects the value of an incremental increase in price while $\lambda^u_{j,t}$ and $\lambda^k_{j,t}$ for j = f,s reflect the value of influencing the fringes' utilisation and capacity decisions. We consider time consistent choices on the part of the dominant producer in that first order conditions for time t decision variables take future decision variables as given.

The first order conditions are given by:

$$(u_{o,t}):\frac{\partial\pi_{o,t}}{\partial u_{o,t}}+\lambda_t^p\frac{\partial P_t}{\partial Q_t}k_{o,t-1}+\underset{t}{E}[\lambda_{t+1}^p\frac{\partial P_{t+1}}{\partial Q_t}k_{o,t-1}]=0 \tag{A.3}$$

$$(k_{o,t}):\frac{\partial\pi_{o,t}}{\partial k_{o,t}}+\underset{t}{E}[\beta\frac{\partial\pi_{o,t+1}}{\partial k_{o,t}}+\lambda_{t+1}^p\frac{\partial P_{t+1}}{\partial Q_{t+1}}u_{o,t+1}+\lambda_{t+2}^p\frac{\partial P_{t+2}}{\partial Q_{t+1}}u_{o,t+1}]=0 \tag{A.4}$$

$$(P_t):\frac{\partial\pi_{d,t}}{\partial p_t}-\lambda_t^p-\lambda_t^{u,s}\frac{\partial^2\pi_{s,t}}{\partial u_{s,t}\partial P_t}-\lambda_t^{u,f}\frac{\partial^2\pi_{f,t}}{\partial u_{f,t}\partial P_t}=0. \tag{A.5}$$

$$(u_{f,t}):\quad\lambda_t^p\frac{\partial P_t}{\partial Q_t}k_{f,t-1}+\underset{t}{E}[\beta\lambda_{t+1}^p\frac{\partial P_{t+1}}{\partial Q_t}k_{f,t-1}]-\lambda_{f,t}^u\frac{\partial^2\pi_{f,t}}{\partial {u_{f,t}}^2}=0. \tag{A.6}$$

$$(k_{f,t}):\quad\underset{t}{E}[\beta\lambda_{t+1}^p\frac{\partial p_{t+1}}{\partial Q_{t+1}}u_{f,t+1}+\beta^2\lambda_{t+2}^p\frac{\partial P_{t+2}}{\partial Q_{t+1}}u_{f,t+1}]-\underset{t}{E}\left[\beta\lambda_{f,t+1}^u\frac{\partial^2\pi_{f,t+1}}{\partial u_{f,t+1}\partial k_{f,t}}\right]$$

$$-\lambda_{f,t}^k\left[\frac{\partial^2\pi_{f,t}}{\partial {k_{f,t}}^2}+\underset{t}{E}\left[\beta\frac{\partial^2\pi_{f,t+1}}{\partial {k_{f,t}}^2}\right]\right]-\underset{t}{E}\left[\beta\lambda_{f,t+1}^k\frac{\partial^2\pi_{f,t+1}}{\partial k_{f,t+1}\partial k_{f,t}}\right]=0. \tag{A.7}$$

$$(u_{s,t}):\quad\lambda_t^p\frac{\partial P_t}{\partial Q_t}k_{s,t-1}+\underset{t}{E}[\beta\lambda_{t+1}^p\frac{\partial P_{t+1}}{\partial Q_t}k_{s,t-1}]-\lambda_{s,t}^u\frac{\partial^2\pi_{s,t}}{\partial {u_{s,t}}^2}=0. \tag{A.8}$$

$$(k_{s,t}):\quad\underset{t}{E}[\beta\lambda_{t+1}^p\frac{\partial P_{t+1}}{\partial Q_{t+1}}u_{s,t+1}+\beta^2\lambda_{t+2}^p\frac{\partial P_{t+2}}{\partial Q_{t+1}}u_{s,t+1}]-\underset{t}{E}\left[\beta\lambda_{s,t+1}^u\frac{\partial^2\pi_{s,t+1}}{\partial u_{s,t+1}\partial k_{s,t}}\right]$$

$$-\lambda_{s,t}^k\left[\frac{\partial^2\pi_{s,t}}{\partial {k_{s,t}}^2}+\underset{t}{E}\left[\beta\frac{\partial^2\pi_{s,t+1}}{\partial {k_{s,t}}^2}\right]\right]-\underset{t}{E}\left[\beta\lambda_{s,t+1}^k\frac{\partial^2\pi_{s,t+1}}{\partial k_{s,t+1}\partial k_{s,t}}\right]=0. \tag{A.9}$$

$$(\lambda_t^p):\quad P(Q_{o,t}+Q_{f,t}+Q_{s,t},Q_{o,t-1}+Q_{f,t-1}+Q_{s,t-1},x_{d,t})-P_t=0 \tag{A.10}$$

where $Q_{j,t}=u_{j,t}k_{j,t-1}$ for $j=o,f,s$.

$$\left(\lambda_{f,t}^u\right):\quad\frac{\partial\pi_{f,t}}{\partial u_{f,t}}=0 \tag{A.11}$$

$$\left(\lambda_{f,t}^k\right):\quad\frac{\partial\pi_{f,t}}{\partial k_{f,t}}+\underset{t}{E}\left[\beta\frac{\partial\pi_{f,t+1}}{\partial k_{f,t}}\right]=0 \tag{A.12}$$

$$\left(\lambda_{s,t}^u\right):\quad\frac{\partial\pi_{s,t}}{\partial u_{s,t}}=0 \tag{A.13}$$

$$\left(\lambda_{s,t}^k\right):\quad\frac{\partial\pi_{s,t}}{\partial k_{s,t}}+\underset{t}{E}\left[\beta\frac{\partial\pi_{s,t+1}}{\partial k_{s,t}}\right]=0 \tag{A.14}$$

In addition to equations (A.3) through (A.14), the model includes equations that describe the dynamic process of the exogenous variables.

$$\text{(demand shifter)}: \quad log(x_{d,t}) = log(x_{b,t}) + log(x_{c,t}) + log(x_{i,t}) \tag{A.15}$$

$$\text{(balanced growth)}: \quad log(x_{b,t}) = log(x_{b,t-1}) + 0.0022 \tag{A.16}$$

$$\text{(cyclical demand)}: \quad log(x_{c,t}) = \eta_y log(WEA_t) \tag{A.17}$$

$$\text{(world IP)}: \quad log(WEA_t) = \theta_{w,1} log(WEA_{t-1}) + \theta_{w,2} log(WEA_{t-2}) \tag{A.18}$$

$$+ \sigma_w e_{w,t}, \quad \text{where } e_{w,t} \sim \text{N}(0,1)$$

$$\text{(shale cost)}: \quad log(z_{s,t}) = log(v_{s,t}^{temp}) + log(v_t^{perm}) - \frac{1}{\eta_{k,s}} log(x_{b,t}) \tag{A.19}$$

$$\text{(temp shale cost)}: \quad log(v_{s,t}^{temp}) = \theta_s log(v_{s,t-1}^{temp}) + \sigma_s e_{s,t}, \quad \text{where } e_{s,t} \sim \text{N}(0,1) \tag{A.20}$$

(perm shale cost) :

$$log(v_t^{perm}) = log(v_{old\ ss}^{perm}), \quad t < 2005\text{Q}1 \tag{A.21}$$

$$log(v_t^{perm}) = log(v_{new\ ss}^{perm}) + 2\rho_{v_s}(log(v_{t-1}^{perm}) - log(v_{new\ ss}^{perm})) \tag{A.22}$$

$$- \rho_{v_s}^2 (log(v_{t-2}^{perm}) - log(v_{new\ ss}^{perm})), \quad t \geq 2005\text{Q}1$$

$$\text{(OPEC Core cost)}: \quad log(z_{o,t}) = log(v_{o,t}^{temp}) - \frac{1}{\eta_{k,o}} log(x_{b,t}) \tag{A.23}$$

$$\text{(temp OPEC Core cost)}: \quad log(v_{o,t}^{temp}) = \theta_o log(v_{o,t-1}^{temp}) + \sigma_o e_{o,t}, \quad \text{where } e_{o,t} \sim \text{N}(0,1) \tag{A.24}$$

$$\text{(conventional cost)}: \quad log(z_{f,t}) = log(v_{f,t}^{temp}) - \frac{1}{\eta_{k,f}} log(x_{b,t}) \tag{A.25}$$

$$\text{(temp conventional cost)}: \quad log(v_{f,t}^{temp}) = \theta_f log(v_{f,t-1}^{temp}) + \sigma_f e_{f,t}, \quad \text{where } e_{f,t} \sim \text{N}(0,1) \tag{A.26}$$

B Elasticity of supply in the short-, medium-, and long-run

For price taking producers, the first order conditions for $u_{j,t}$ and $k_{j,t}$ are given by:

$$P_t - c_{1,j}\left(\frac{1+\frac{1}{\eta_{u,j}}}{1+\frac{1}{\eta_{k,j}}}\right) u_{j,t}^{\frac{1}{\eta_{u,j}}} z_{j,t} k_{j,t-1}^{\frac{1}{\eta_{k,j}}} = 0 \tag{B.1}$$

and

$$\begin{aligned}
&-\kappa_j\left(\frac{k_{j,t}/x_{b,t}}{k_{j,t-1}/x_{b,t-1}}-1\right)\frac{x_{b,t-1}}{x_{b,t}}\frac{z_{j,t}k_{j,t-1}^{\frac{1}{\eta_{k,j}}}}{1+\frac{1}{\eta_{k,j}}}\\
&+\beta \underset{t}{E}\Bigg[P_{t+1}u_{j,t+1}-\left(C_j(u_{j,t},k_{j,t-1})+\frac{\kappa_j}{2}\left(\frac{k_{j,t+1}/x_{b,t+1}}{k_{j,t}/x_{b,t}}-1\right)^2\right)z_{j,t+1}k_{j,t}^{\frac{1}{\eta_{k,j}}}\\
&+\kappa_j\left(\frac{k_{j,t+1}/x_{b,t+1}}{k_{j,t}/x_{b,t}}-1\right)\left(\frac{k_{j,t+1}/x_{b,t+1}}{k_{j,t}/x_{b,t}}\right)\frac{z_{j,t+1}k_{j,t}^{\frac{1}{\eta_{k,j}}}}{1+\frac{1}{\eta_{k,j}}}\Bigg]=0
\end{aligned} \tag{B.2}$$

Recall output is given by:

$$Q_{j,t} = u_{j,t}k_{j,t-1} \tag{B.3}$$

In the short-run, $k_{j,t-1}$ is predetermined and the elasticity of supply is $\frac{\partial u_{j,t}}{\partial P_t}/\frac{u_{j,t}}{P_t}$. Using (B.1), the price elasticity of supply in the current period is $\eta_{u,j}$. In the long-run (steady state), the two first order conditions imply:

$$P - c_{1,j}\left(\frac{1+\frac{1}{\eta_{u,j}}}{1+\frac{1}{\eta_{k,j}}}\right) u_j^{\frac{1}{\eta_{u,j}}} z_j k_j^{\frac{1}{\eta_{k,j}}} = 0 \tag{B.4}$$

and

$$P\ u_j - \left(c_{0,j} + c_{1,j}u_j^{1+\frac{1}{\eta_{u,j}}}\right) z_j k_j^{\frac{1}{\eta_{k,j}}} = 0 \tag{B.5}$$

We normalize $u_j = 1$ in the steady state, which in turn implies $c_{0,j} = 1 - c_{1,j}$ and $c_{1,j} = \frac{1+\frac{1}{\eta_{k,j}}}{1+\frac{1}{\eta_{u,j}}}$. Output in the steady state is determined by:

$$Q_j = k_j = (P/z_j)^{\eta_{k,j}} \tag{B.6}$$

The effect on output of a permanent change in prices is then:

$$\Delta log(Q_j) = \eta_{k,j} \Delta log(P) \tag{B.7}$$

with the elasticity of supply $\eta_{k,j}$.

For the medium run elasticity, we log-linearize equations (B.1) and (B.2) around the steady state:

$$\hat{p}_{t+1} = \left(\frac{1}{\eta_{u,j}} \hat{u}_{j,t} + \hat{z}_{j,t} + \frac{1}{\eta_{k,j}} \hat{k}_{j,t-1} \right) = 0 \tag{B.8}$$

$$\begin{aligned} &- \frac{\kappa_j}{\left(1 + \frac{1}{\eta_{k,j}}\right) \gamma_b} \left(\hat{k}_{j,t} - \hat{k}_{j,t-1} \right) \\ &+ \beta \left(\hat{p}_{t+1} - \frac{1}{\eta_{k,j}} \hat{u}_{j,t+1} - \hat{z}_{j,t+1} - \frac{1}{\eta_{k,j}} \hat{k}_{j,t+1} - \frac{\kappa_j}{1 + \frac{1}{\eta_{k,j}}} \left(\hat{k}_j, t - \hat{k}_{j,t-1} \right) \right) = 0 \end{aligned} \tag{B.9}$$

where $\gamma_x = x_{b,t}/x_{b,t-1}$ is the growth rate of the balanced growth trend. Combining equations (B.8) ad (B.9), yields a second order difference equation for $\hat{k}_{j,t}$

$$\eta_{k,t} \left(\hat{p}_{t+1} - \hat{z}_{j,t+1} \right) - (1 + \theta_{1,j} + \theta_{2,j}) \hat{k}_{j,t} + \theta_{1,j} \hat{k}_{j,t-1} + \theta_{2,j} \hat{k}_{j,t+1} = 0, \tag{B.10}$$

where $\theta_{1,j} = \frac{\kappa_j}{\left(1+\frac{1}{\eta_{k,j}}\right)\beta\gamma_b\left(\eta_{k,j}-\eta_{u,j}\right)}$ and $\theta_{2,j} = \beta\gamma_b\theta_{1,j}$. Denote the two roots of this difference equation $\phi_{1,j}$ and $\phi_{2,j}$ with $|\phi_{1,j}| < 1$ and $|\phi_{2,j}| > 1$. We can rewrite (B.10) as

$$\eta_{k,t} \left(\hat{p}_{t+1} - \hat{z}_{j,t+1} \right) - \theta_{2,j} \phi_{2,j} \left(1 - \phi_{2,j}^{-1} L^{-1} \right) \left(1 - \phi_{1,j} L \right) \hat{k}_{j,t} = 0, \tag{B.11}$$

where L is lag operator, $Lk_{j,t} = k_{j,t-1}$. Solving (B.11) forwards yields:

$$\hat{k}_{j,t} = \frac{\eta_{k,j}}{\theta_{2,j}\phi_{2,j}} \sum_{i=0}^{\infty} \phi_{2,j}^{-i} \left(\hat{p}_{t+i} - \hat{z}_{j,t+i}\right) + \phi_{1,j}\hat{k}_{j,t-1}. \tag{B.12}$$

Note that total production (relative to steady state) is:

$$\hat{q}_{j,t+i} = \eta_{k,j} \left(\hat{p}_{t+i} - \hat{z}_{j.t+i}\right) + \left(1 - \frac{\eta_{u,j}}{\eta_{k,j}}\right) \hat{k}_{j,t+i-1} \tag{B.13}$$

Consider the case of a permanent one-time increase in the price level so that $\hat{p}_{t+i} = 0$ and $\hat{z}_{j,t+i} = 0$. Taking the existing capacity at t-1 as given, from equation (B.7), $\hat{k}_{j,t-1} = -\eta_{k,j}\Delta log(P)$. Equation (B.12) implies:

$$\hat{k}_{j,t+i} = -\phi_{1,j}^{i+1}\eta_{k,j}\Delta log(P) \tag{B.14}$$

and from equation (B.13)

$$\hat{q}_{j,t+i} = -\left(1 - \frac{\eta_{u,j}}{\eta_{k,j}}\right) \phi_{1,j}^{i}\eta_{k,j}\Delta log(P) \tag{B.15}$$

Adding the the change in steady state log output to $\hat{q}_{j,t+i}$ implies

$$log(Q_{j.t+i}) - log(Q_{j,old\ ss}) = \Delta log(Q_{ss}) + \hat{q}_{j,t+i} \tag{B.16}$$

which in turn implies

$$log(Q_{j.t+i}) - log(Q_{j,old\ ss}) = \left(\eta_{k,j}\left(1 - \phi_{1,j}^{i}\right) + \eta_{u,j}\phi_{1,j}^{i}\right) \Delta log(P) \tag{B.17}$$

Thus, the elasticity of supply for a permanent increase in the price level is essentially a weighted average of the short-run elasticity of supply, $\eta_{u,j}$ and the long-run elasticity of supply $\eta_{k,j}$. The weight on the long-run elasticity of supply increases, the further we are in the future. The adjustment cost parameter, κ_j, affects the weight as well, with $\frac{\partial \phi_{1,j}}{\partial \kappa_j} > 0$.

Figure B1 displays how the the medium-run elasticities depend on the adjustment cost parameters as well as on the short- and long-run supply elasticities. Panel A sets the values of η_u and η_k to be the mode of the prior distribution for these parameters for conventional fringe producers in our empirical model. Panel B sets the values of η_u and η_k to be the mode of the prior distribution for shale and OPEC core.

As the medium-run supply elasticity depends on the horizon, the short- and long-run elasticities of supply and the adjustment cost parameter, κ_j, the prior distribution of the medium-run supply elasticities depend on the prior distributions of these parameters. In order to get a sense of the prior distribution for the medium-run elasticities, we randomly draw these parameters from their prior distributions and calculate the medium-run supply elasticities for various horizons. Figure B2 displays a box chart of the implied prior distribution for medium-run supply elasticities for selected horizons for conventional fringe producers and shale and OPEC core (assuming OPEC Core acts a price taker). Figure B3, displays the box plot for the posterior distribution of the medium-run elasticities.

Figure B1: Medium-run supply elasticities for various adjustment cost parameters

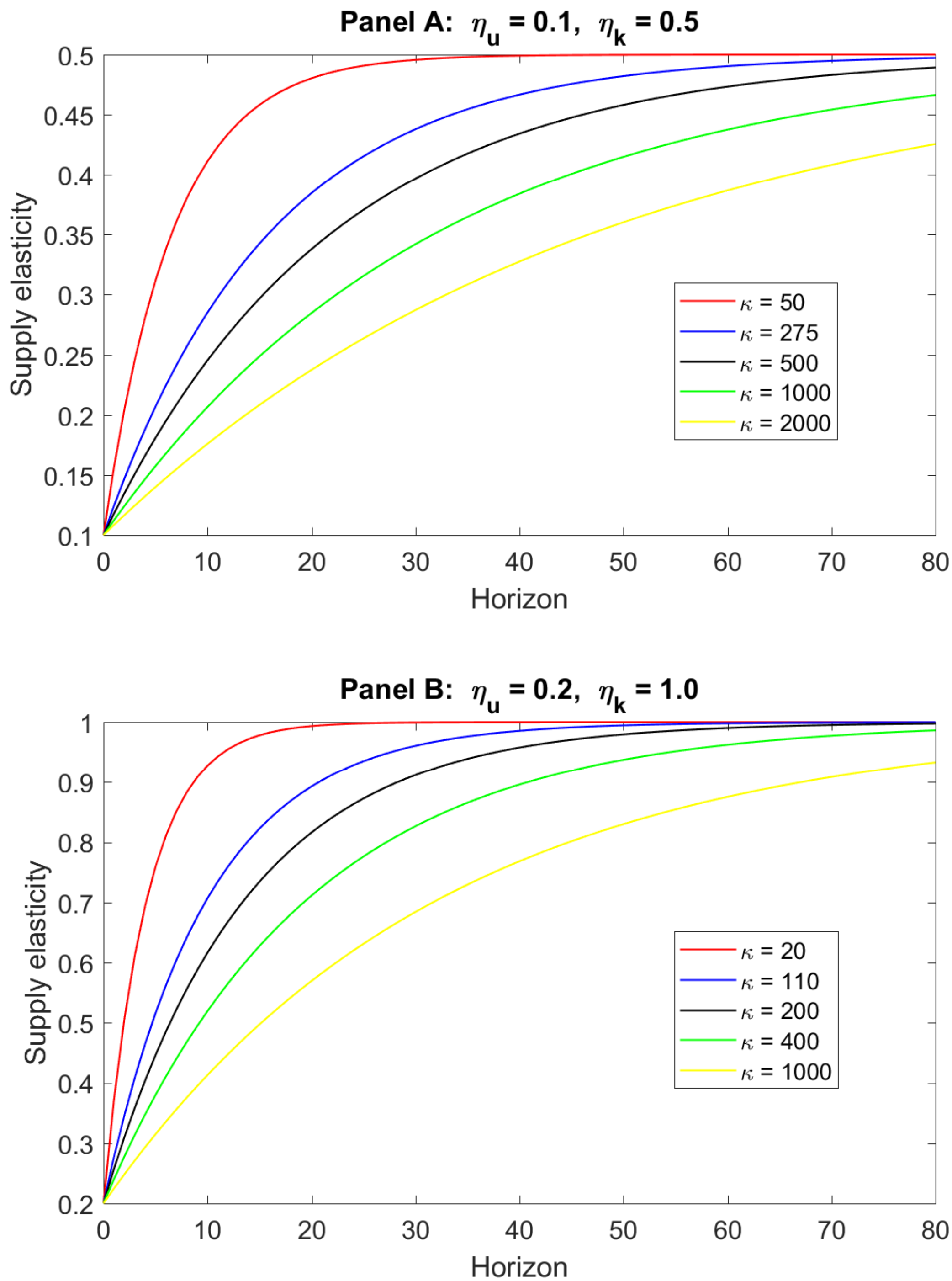
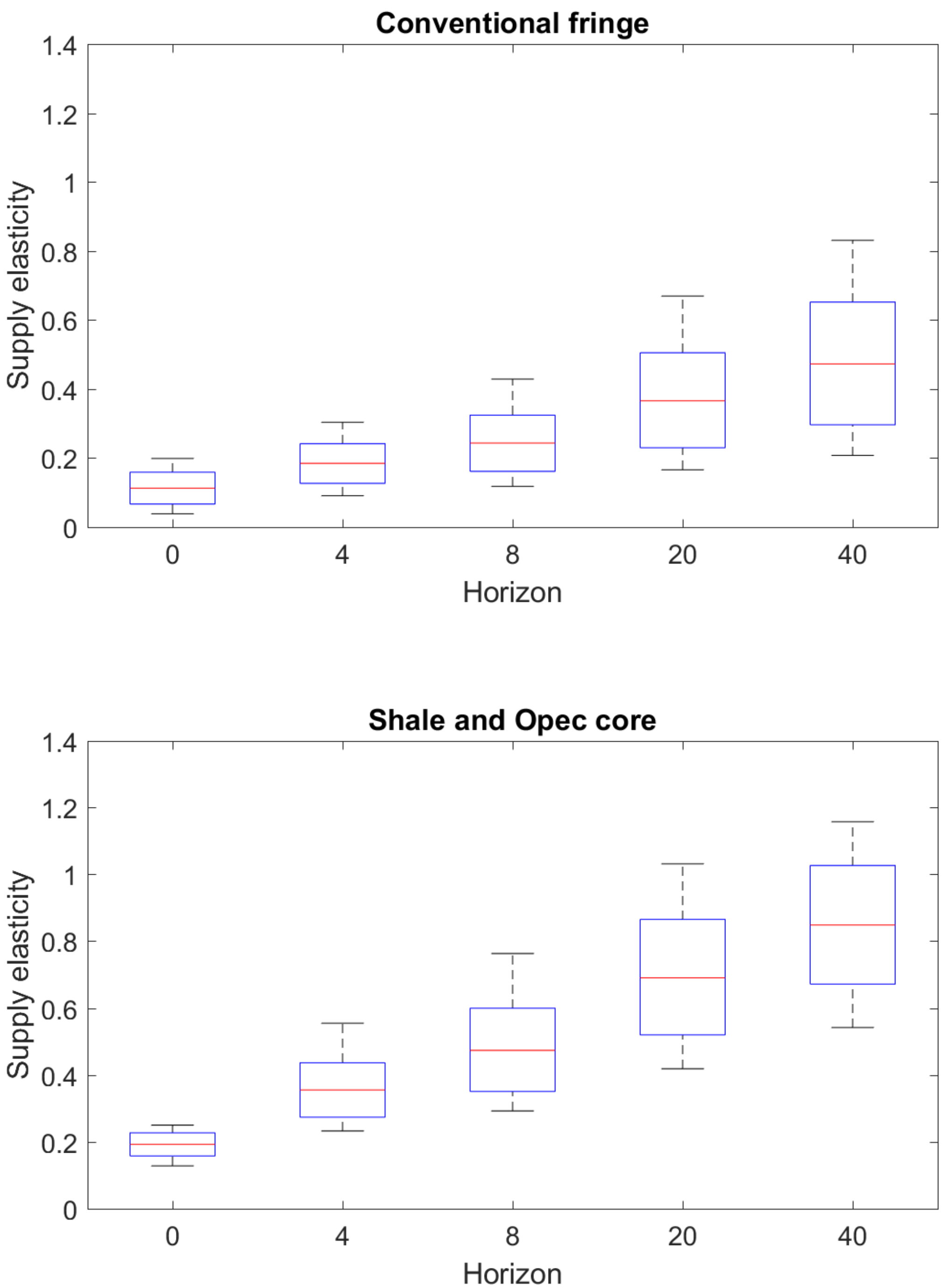
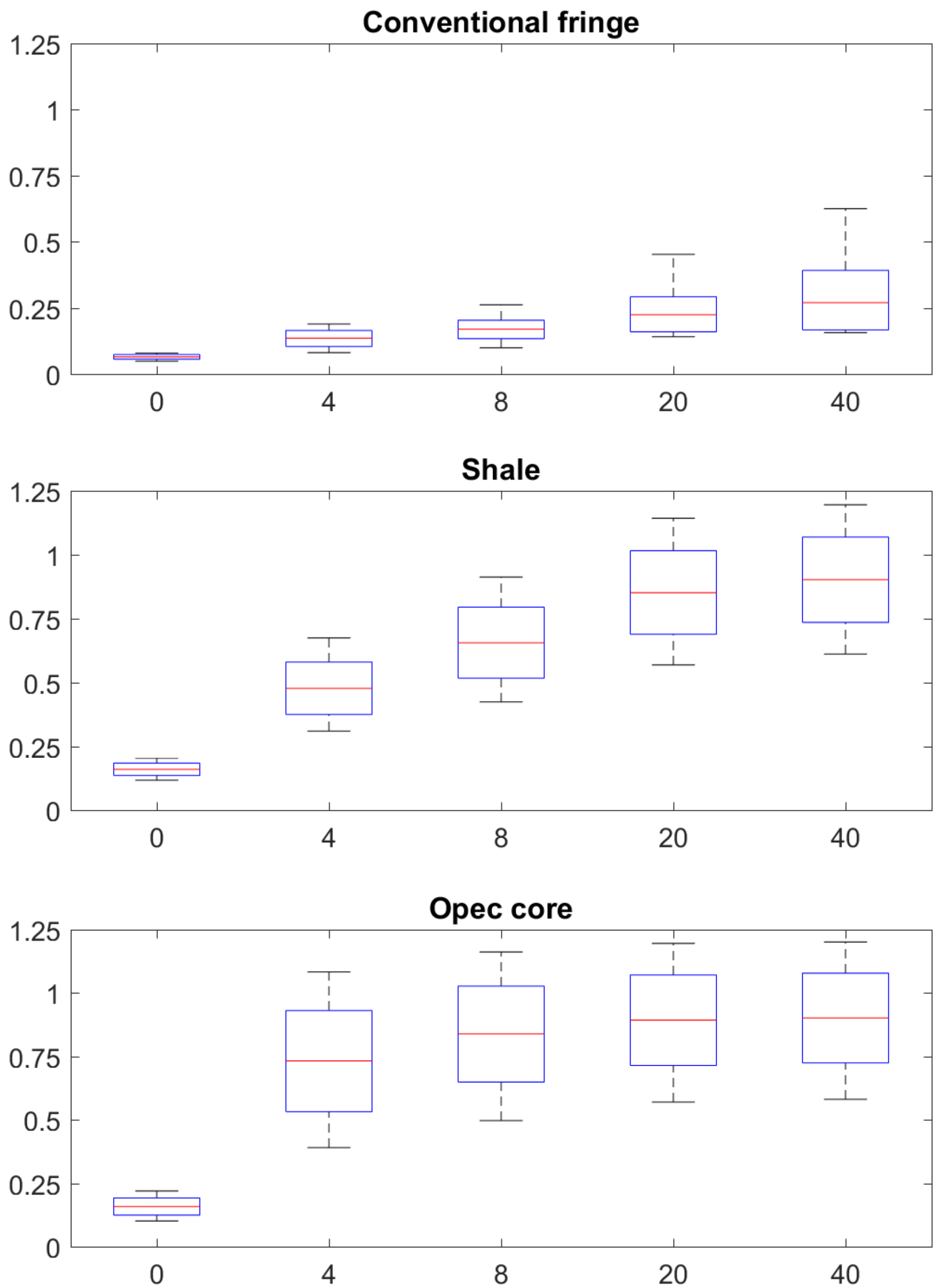


Figure B2: Prior distribution for Medium-run supply (cost) elasticities



Note: The centre line in the box is the mean of the posterior distribution. The bottom and top of the box are the 25th and 75th percentiles. The tick marks at the end of the vertical line represent the 5th and 95th percentiles.

Figure B3: Posterior distribution for Medium-run supply (cost) elasticities



Note: The centre line in the box is the mean of the posterior distribution. The bottom and top of the box are the 25th and 75th percentiles. The tick marks at the end of the vertical line represent the 5th and 95th percentiles.

C Model solution

The model outlined in equations (A.3)-(A.26) can be written as:

$$\underset{t}{E}\left[g(X_t, X_{t+1}, X_{t-1}, e_t, v^{perm}_{old\ ss}, \theta)\right] = 0, \quad t < 2005Q1 \tag{C.1}$$

and

$$\underset{t}{E}\left[g(X_t, X_{t+1}, X_{t-1}, e_t, v^{perm}_{new\ ss}, \theta)\right] = 0, \quad t \geq 2005Q1 \tag{C.2}$$

where X_t are the endogenous variables in the system, e_t a vector of exogenous shocks, $v^{perm}_{old\ ss}$ is the steady state of shale producer's production cost before the shale revolution, and $v^{perm}_{new\ ss}$ is the steady state of shale producer's production-cost variable after the shale revolution. A first order approximation around a steady state yields the difference-equation system of the form:

$$\begin{aligned} A(X_{ss|v^{perm}_{ss}})\underset{t}{E}(X_{t+1} - X_{ss|v^{perm}_{ss}}) + B(X_{ss|v^{perm}_{ss}})(X_t - X_{ss|v^{perm}_{ss}}) \\ + C(X_{ss|v^{perm}_{ss}})(X_{t-1} - X_{ss|v^{perm}_{ss}}) + D(X_{ss|v^{perm}_{ss}})e_t = 0 \end{aligned} \tag{C.3}$$

$X_{ss|v^{perm}_{ss}}$ is the steady state value of the variables in the model which is, in turn, a function of the structural parameters of the model (θ) and the steady-state value of shale producer's costs. The rational expectations solution to this difference-equation system will have the form:

$$X_t = G(X_{ss|v^{perm}_{ss}}) + P(X_{ss|v^{perm}_{ss}})X_{t-1} + Q(X_{ss|v^{perm}_{ss}})e_t \tag{C.4}$$

where

$$G(X_{ss|v^{perm}_{ss}}) = X_{ss|v^{perm}_{ss}} - P(X_{ss|v^{perm}_{ss}})X_{ss|v^{perm}_{ss}}.$$

For time periods before the shale revolution, we linearly approximate the model around the pre-shale steady state:

$$X_t = G^{[0]} + P^{[0]}X_{t-1} + Q^{[0]}e_t \tag{C.5}$$

where where $G^{[0]}$ is a $n \times 1$ vector, $P^{[0]}$ is $n \times n$ matrix, and $Q^{[0]}$ is a $n \times p$ matrix. The matrices $G^{[0]}$, $P^{[0]}$, and $Q^{[0]}$ depend on the steady-state values of the endogenous variables when shale production costs are equal $v^{perm}_{old\ ss}$ (and the other structural parameters θ):

$$\begin{aligned} G^{[0]} &= G(X_{ss|v^{perm}_{old\ ss}}) \\ P^{[0]} &= P(X_{ss|v^{perm}_{old\ ss}}) \\ Q^{[0]} &= Q(X_{ss|v^{perm}_{old\ ss}}) \end{aligned} \tag{C.6}$$

These matrices depend on the steady state values of X_t when the steady-state value of shale production cost is equal to $v^{perm}_{old\ ss}$ or $X_{ss|v^{perm}_{old\ ss}}$. Equation C.5 holds in all the time periods before 2005Q1 and reflects the fact that the 'shale revolution' was a surprise from the point of view of time periods before 2005Q1.

For the time periods after the shale revolution begins, our approach is similar to the piece-wise linear approximation of Guerrieri and Iacoviello (2015). We start at the post-shale steady state and work backwards in time. For a time period sufficiently far in the future, we approximate the model around the new steady state. That is, for $t \geq t_N$:

$$X_t = G^{[N]} + P^{[N]} X_{t-1} + Q^{[N]} e_t \tag{C.7}$$

where

$$\begin{aligned} G^{[N]} &= G(X_{ss|v^{perm}_{new\ ss}}) \\ P^{[N]} &= P(X_{ss|v^{perm}_{new\ ss}}) \\ Q^{[N]} &= Q(X_{ss|v^{perm}_{new\ ss}}) \end{aligned} \tag{C.8}$$

$G^{[N]}$ is a $n \times 1$ vector, $P^{[N]}$ is $n \times n$ matrix, and $Q^{[N]}$ is a $n \times p$ matrix.

For periods before t_N, the new steady state is not a good approximation, we approximate around a different value, $v^{perm}_{t_N}$, where $v^{perm}_{t_N}$ is value of the transition variable in time period

t_N. The resulting first order approximation for time periods, $t_{N-1} \leq t < t_N$, is given by:

$$A^{[t_N]} \underset{t}{E}(X_{t+1} - X_{ss|v_{t_N}^{perm}}) + B^{[t_N]}(X_t - X_{ss|v_{t_N,\theta}^{perm}}) \tag{C.9}$$
$$+C^{[t_N]}(X_{t-1} - X_{ss|v_{t_N}^{perm}}) + D^{[t_N]}e_t + E^{[t_N]} = 0$$

where

$$\begin{aligned} A^{[t_N]} &= A(X_{ss|v_{t_N}^{perm}}) \\ B^{[t_N]} &= B(X_{ss|v_{t_N}^{perm}}) \\ C^{[t_N]} &= C(X_{ss|v_{t_N}^{perm}}) \\ D^{[t_N]} &= D(X_{ss|v_{t_N}^{perm}}) \\ E^{[t_N]} &= E(X_{ss|v_{t_N}^{perm}}) \end{aligned} \tag{C.10}$$

The value $X_{ss|v_{t_N}^{perm}}$ represents the steady-state value of X_t for the model where the steady-state value of $v_t^{perm} = v_{t_N}^{perm}$. Given the actual model implies a steady-state value of $v_t^{perm} = v_{new\ ss}^{perm}$, the constant term in equation (C.9), $E^{[t_N]} = E(X_{ss|v_{t_N}^{perm}})$, reflects the fact that $v_{t_N}^{perm} \neq v_{new\ ss}^{perm}$. Recall that given equation (C.2), $g(X_{ss|v_{new\ ss}^{perm}}, X_{ss|v_{new\ ss}^{perm}}, X_{ss|v_{new\ ss}^{perm}}, 0, v_{new\ ss}^{perm}, \theta) = 0$ in the new steady state. When not evaluating the function at the new steady state, the term $E^{[t_N]} = E(X_{ss|v_{t_N}^{perm}}) = g(X_{ss|v_{t_N}^{perm}}, X_{ss|v_{s,t_N}^{perm}}, X_{ss|v_{t_N}^{perm}}, 0, v_{new\ ss}^{perm}, \theta) \neq 0$.

Combining equations (C.7) and (C.9), we get for $t_{N-1} \leq t < t_N$:

$$X_t = G^{[t]} + P^{[t]}X_{t-1} + Q^{[t]}e_t \tag{C.11}$$

where

$$G^{[t]} = -\left(A^{[t_N]}P^{[t+1]} + B^{[t_n]}\right)^{-1} \tag{C.12}$$
$$\left(E^{[t_N]} + A^{[t_N]}G^{[t+1]} - \left(A^{[t_N]} + B^{[t_N]} + C^{[t_N]}\right) X_{ss|v_{t_N}^{perm}}\right)$$

$$P^{[t]} = -\left(A^{[t_N]} P^{[t+1]} + B^{[t_N]}\right)^{-1} C^{[t_N]} \tag{C.13}$$

$$Q^{[t]} = -\left(A^{[t_N]} P^{[t+1]} + B^{t_N]}\right)^{-1} D^{[t_N]} \tag{C.14}$$

One can iterate equations (C.11)-(C.14) backwards allowing for the approximation point on the transition path, v^{perm}_{s,t_i} to change. In our application, we set $t_N = 2045Q1$, $t_{N-1} = 2035Q1$, and $t_{N-2} = 2030Q1$. From 2030Q1 until 2005Q3, we work backwards taking every second quarter of $v^{perm}_{t_i}$ as the approximation point.

D Robustness Analysis

In this appendix we explore several alternative modelling assumptions. In particular, we examine alternative assumptions about the size of future shale production and the use of OPEC rather than OPEC Core as the dominant oil producer in the empirical model.

D.1 Alternative future Shale steady states

In our benchmark model, we assume that the future steady-state shale market share was 20%. Here we consider two alternative steady states, one where shale's share is 15% and another where shale's share is 25%. Taking the estimated posterior distribution for the model with a shale steady state of 20%, we examine the implications of changing shale's steady-state share to 15% or 25%. Table 7 in the text displays the deterministic steady-state price, output, and market shares. Not surprisingly, as shale's share rises, the market price of oil falls. Interestingly, OPEC Core's share is relatively stable even in the steady state when shale's share rises.

Figure D1 and D2 display the implied transition path for the model. Within the sample, there is virtually no difference between the model with shale share of 15% and 25%. As the shale transition progresses into the future, the differential effects on price, output, OPEC Core, and shale share of different steady-state shale shares are more pronounced. Tables D1 and D2 display the variances and variance decomposition for the two alternative assumptions about shale's long-run share. From Table 6 in the text and Tables D1 and D2 in this appendix, the shale revolution results in the reduction of volatility in oil prices and output across all three alternative assumptions about the future size of the shale sector. Although the reduction in volatility in the long-run is larger in the models with a larger shale sector, the reduction in volatility during the transition period (2021Q3) is of similar magnitude across the three models. Figures D3 and D4 display the relative conditional forecast variance for select horizons. Again, for both shale shares, oil price variability declines over the transition period, with the scenario with a larger shale share displaying a larger decline in volatility.

D.2 OPEC vs OPEC Core

This section presents the results if we estimate the model using OPEC instead of OPEC Core as the dominant producer. Tables D3 and D4 present the posterior distributions of the structural parameters and implied elasticities of supply for the model with OPEC. For the most part, the short-run, medium-run, and long-run supply elasticities for the model using OPEC were similar to those for the model with OPEC Core. The demand elasticity is higher in the model with OPEC. In the steady state, in order for the Stackelberg price to be a markup over marginal cost, the market share of the dominant producer puts constraints on how low the price elasticity for the dominant producer's demand can get.[1] Given OPEC market share is greater than OPEC Core's market share, this constraint on the elasticity of demand is more binding for the model with OPEC.

Table D5 displays the steady states implied by this model. For the model with OPEC, the effect on the steady-state price is somewhat smaller that the model with OPEC Core (40% decline versus 46% decline) while the increase in output is nearly double (22% versus 11%). As in the model with OPEC Core, the model predicts very little change in OPEC's market share. Again, the dominant producer reacts to the shale revolution by effectively keeping its market share constant. The implied steady-state price to marginal cost ratio for the model with OPEC is substantially larger than for the model with OPEC Core–the OPEC model having a posterior distribution centred around 15 as compared to 11 for the OPEC Core model. In the text, we argued that the OPEC Core model's price-marginal cost ratio was consistent with actual price and cost estimates of OPEC Core producers. That is a harder case to make for the OPEC model. Asker et al. (2019) find that many non-core OPEC members have price-marginal cost ratios that are substantially lower than those of

[1] Recall $\eta_d \frac{(1-\rho_d)}{1-\beta\rho_d} + \eta_{u,s} s_s + \eta_{u,f} s_f$ is the steady state elasticity of demand for the dominant producer. This elasticity must be greater that the dominant producer's market share in order for the price markup to be positive.

OPEC Core producers and substantially lower than the price-marginal cost ratios implied by the all-of-OPEC model. Our interpretation of these results is that the dominant producer, competitive fringe framework applies less well to OPEC than it does to OPEC Core.

Figure D6 displays the relative decline in the forecast error variance along the transition path relative to pre-shale variance. Both suggest that the forecast error variance falls rapidly along the transition before the relative decline levels out and at longer horizons, begins to rise as shale's share increases. Table D6 displays the forecast error decomposition for log real oil price. As in the OPEC Core model, the contribution of conventional fringe shocks diminish as shale's share increases. On the other hand, shale's contribution to log real price forecast error variance increases dramatically at longer horizons and as shale's share increases. Despite the dominant producers' share is larger in the version of the model with OPEC, OPEC costs shocks are a small contributor to forecast error variance of prices.

Figure D7 displays the oil market responses to a supply shock to the conventional fringe that lasts 8 quarters. The shock is meant to mimic the supply disruption brought about by the Russian invasion of Ukraine.[2] Starting in 2022Q1, The model suggests that the shock would lead to an increase of 4% in oil prices by the second quarter, a fall of 1.3% in global oil production, an increase of 1.1% in shale output. The full OPEC model suggests a 0.7% fall in OPEC production. These numbers are quantitatively lower than those of the OPEC Core benchmark model and are smaller than the response of actual prices and output in 2022Q2.

[2] The size of the shock is set so that there is a decline in conventional fringe supply of 2.8% if price remained unchanged.

Table D1. Conditional forecast variance decompositions of log real oil price: Model with OPEC core and shale steady state share = 15%
(mean of posterior distribution)

Panel A: pre-shale steady state

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0196	46.7	33.2	17.6	0.0	2.5
1 year	0.1001	29.1	32.7	36.6	0.0	1.6
2 year	0.1614	27.2	27.2	44.4	0.0	1.2
5 year	0.2962	26.5	18.1	54.7	0.0	0.7
10 year	0.4502	27.8	13.5	58.1	0.0	0.5

Panel B: Transition period (2021Q3)

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0182	47.4	33.7	15.0	0.7	3.2
1 year	0.0883	30.2	34.0	32.2	1.5	2.2
2 year	0.1384	28.2	28.6	39.2	2.3	1.7
5 year	0.2454	27.3	19.2	48.6	3.7	1.1
10 year	0.3678	28.4	14.4	52.0	4.5	0.8

Panel C: post-shale steady state

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0166	46.9	33.3	11.1	3.6	4.3
1 year	0.0762	30.3	34.2	23.7	7.4	3.5
2 year	0.1138	27.9	28.8	28.3	11.4	2.8
5 year	0.2008	26.1	19.4	34.6	17.3	1.8
10 year	0.2939	36.6	14.6	37.4	19.3	1.3

Table D2. Conditional forecast variance decompositions of log real oil price: Model with OPEC core and shale steady state share = 25% (mean of posterior distribution)

Panel A: pre-shale steady state

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0194	46.4	33.0	17.4	0.0	2.5
1 year	0.0993	28.9	32.4	36.4	0.0	1.6
2 year	0.1603	27.0	27.0	44.1	0.0	1.2
5 year	0.2940	26.3	17.9	54.3	0.0	0.7
10 year	0.4469	27.6	13.4	57.7	0.0	0.5

Panel B: Transition period (2021Q3)

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0174	47.1	33.5	13.3	1.8	3.6
1 year	0.0825	30.2	33.9	28.8	3.6	2.7
2 year	0.1282	28.1	28.5	35.0	5.6	2.1
5 year	0.2254	26.8	19.2	43.3	8.7	1.3
10 year	0.3368	27.6	14.2	46.5	10.0	1.0

Panel C: post-shale steady state

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0159	45.3	32.2	7.5	9.1	5.1
1 year	0.0715	28.8	32.6	15.7	17.3	4.9
2 year	0.1106	25.6	26.7	18.0	25.1	3.9
5 year	0.1941	22.8	17.6	21.3	35.0	2.5
10 year	0.2848	23.1	13.3	23.6	37.4	1.8

Table D3. Posterior distribution of parameters: Model with OPEC

	structural parameters	mode	mean	5th	95th
	demand elasticities:				
1.	long-run demand elasticity (-η_d)	-0.40	-0.39	-0.41	-0.37
2.	short-run demand elasticity (-$(1-\rho_d)\eta_d$)	-0.29	-0.29	-0.30	-0.29
3.	oil demand elast. wrt world econ. activ. (η_y)	1.92	1.96	1.56	2.35
	long-run supply elasticities:				
4.	Conventional fringe ($\eta_{k,f}$)	0.26	0.37	0.18	0.74
5.	OPEC ($\eta_{k,o}$)	0.95	0.91	0.59	1.21
6.	Shale fringe ($\eta_{k,s}$)	0.98	0.90	0.60	1.20
	short-run supply elasticities:				
7.	Conventional fringe ($\eta_{u,f}$)	0.06	0.07	0.05	0.08
8.	OPEC ($\eta_{u,o}$)	0.18	0.18	0.12	0.24
9.	Shale fringe ($\eta_{u,s}$)	0.18	0.18	0.13	0.22
	adjustment costs:				
10.	Conventional fringe (κ_f)	1802.71	1705.82	604.68	3306.81
11.	OPEC (κ_o)	6.50	25.55	1.95	83.62
12.	Shale fringe (κ_s)	56.79	68.31	18.49	157.95

	shock parameters	mode	mean	5th	95th
13.	AR(1) coeff. for oil specific demand shock	0.99	0.99	0.98	0.999
14.	AR(1) coeff. for OPEC cost shock	0.97	0.98	0.94	0.999
15.	AR(1) coeff. for conv. fringe cost shock	0.95	0.97	0.94	0.996
16.	AR(1) coeff. for shale cost shock	0.98	0.99	0.96	0.999
17.	AR(1) coeff. for world IP shock	1.21	1.20	1.06	1.34
18.	AR(2) coeff. for world IP shock	-0.26	-0.25	-0.39	-0.11
19.	shale cost transition parameter (ρ_{v_s})	0.95	0.95	0.94	0.96
20.	std. dev. for oil specific demand shock	0.042	0.040	0.036	0.045
21.	std. dev. for OPEC cost shock	0.26	0.29	0.20	0.40
22.	std. dev. for conv. fringe cost shock	0.27	0.25	0.21	0.31
23.	std. dev. for shale cost shock	0.20	0.22	0.16	0.29
24.	std. dev. for world IP shock	0.01	0.01	0.01	0.02

Table D4. Implied supply elasticities: Model with OPEC (mean of posterior distribution)

Panel A: pre-shale steady state

horizon	market	conv. fringe	shale fringe	OPEC
initial quarter	0.055	0.066	0.178	0.039
1 year	0.101	0.085	0.304	0.122
2 year	0.174	0.141	0.588	0.219
5 year	0.283	0.251	0.850	0.323
10 year	0.362	0.334	0.899	0.398

Panel B: transition period (2021Q3)

horizon	market	conv. fringe	shale fringe	OPEC
initial quarter	0.086	0.066	0.178	0.080
1 year	0.151	0.084	0.296	0.185
2 year	0.285	0.140	0.583	0.352
5 year	0.450	0.253	0.852	0.514
10 year	0.546	0.334	0.901	0.604

Panel C: post-shale steady state

horizon	market	conv. fringe	shale fringe	OPEC
initial quarter	.102	0.066	0.178	0.101
1 year	0.178	0.083	0.294	0.218
2 year	0.334	0.141	0.586	0.408
5 year	0.493	0.254	0.854	0.560
10 year	0.554	0.332	0.897	0.613

Table D5. Posterior distribution of post-shale steady states
Model with OPEC

Panel A: Steady-state price, output, and market share

	variable	pre-shale steady state	**post-shale steady state** mode	mean	5th	95th
1.	real oil price	100.0	56.9	60.5	53.0	70.7
2.	market oil output	100.0	125.0	122.0	114.7	127.7
3.	OPEC share	40.0	38.8	38.9	37.5	40.3
4.	Shale share	0.5	20.0	20.0	20.0	20.0
5.	Conventional Fringe share	59.5	41.1	41.1	39.7	42.5

Panel B: Price to marginal cost ratio for OPEC

		mode	mean	5th	95th
1.	pre-shale steady state	15.0	15.4	10.2	22.7
2.	post-shale steady state	6.9	7.6	5.3	11.0

Table D6. Conditional forecast variance decompositions of log real oil price: Model with OPEC (mean of posterior distribution)

Panel A: pre-shale steady state

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0211	64.5	26.6	6.7	0.0	2.3
1 year	0.0681	53.6	32.2	11.8	0.0	2.9
2 year	0.1050	52.0	29.0	16.0	0.0	3.0
5 year	0.1885	50.5	21.3	25.3	0.0	2.9
10 year	0.2837	50.9	16.0	30.3	0.0	2.7

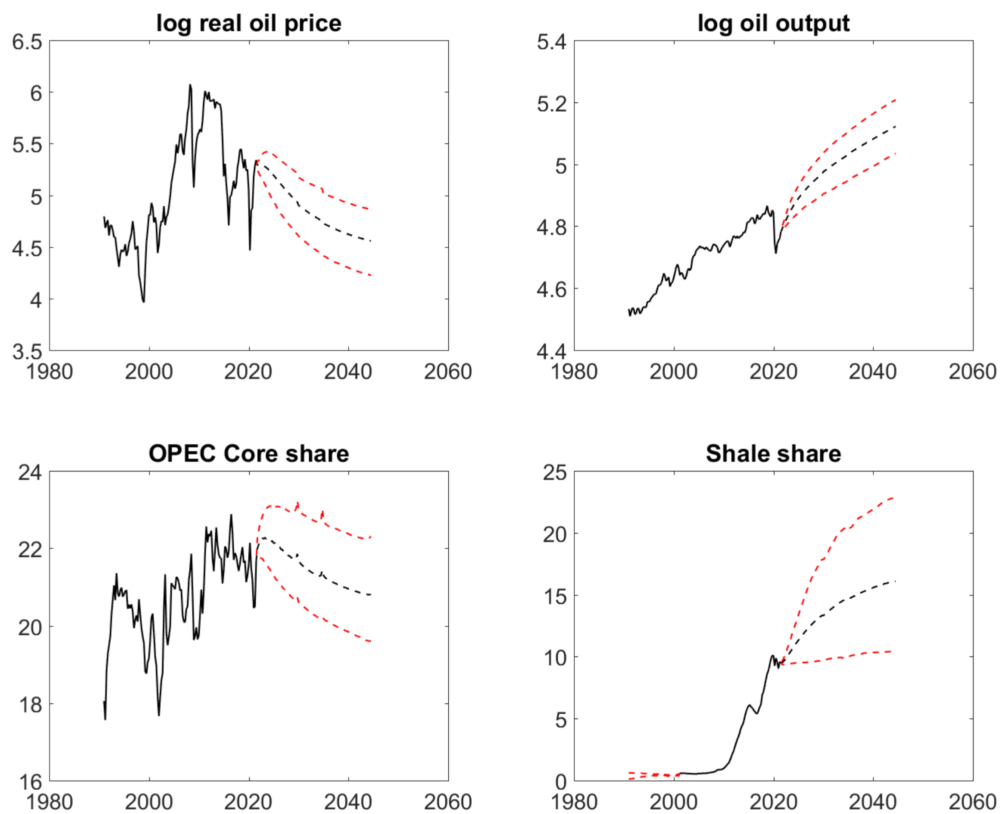
Panel B: Transition period (2021Q3)

		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0181	63.6	27.4	3.8	1.5	3.7
1 year	0.0573	51.5	31.0	6.2	6.2	5.1
2 year	0.0872	48.5	27.2	8.6	10.4	5.3
5 year	0.1562	44.9	19.4	13.6	17.1	5.0
10 year	0.2380	43.8	14.2	16.7	20.6	4.6

Panel C: post-shale steady state

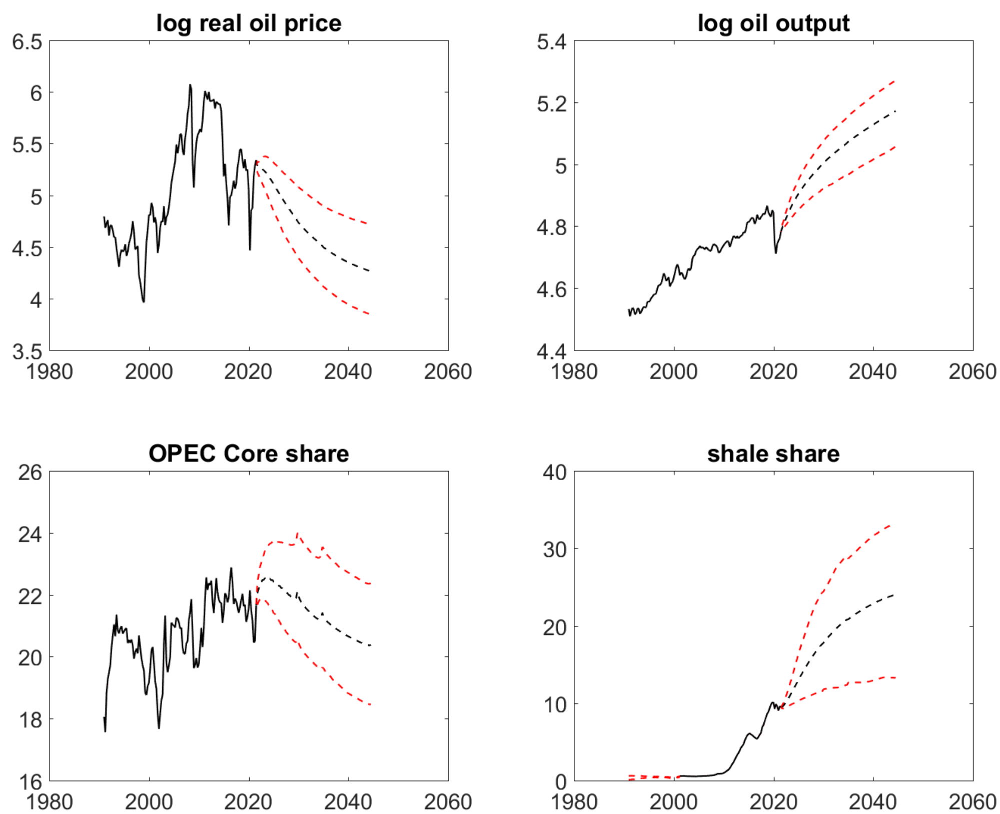
		Percent contribution of shocks to:				
horizon	variance	oil specific demand	world demand	Conv. supply	shale supply	OPEC core supply
initial quarter	0.0170	62.2	27.3	2.6	3.3	4.6
1 year	0.0553	47.9	28.9	3.8	12.6	6.8
2 year	0.0862	43.2	24.4	5.0	20.3	6.9
5 year	0.1610	37.6	16.5	7.6	31.9	6.3
10 year	0.2515	35.8	11.8	9.3	37.2	5.8

Figure D1: Oil market and shale transition: Shale share = 15%
Mean, 5th and 95th percentiles of posterior distribution



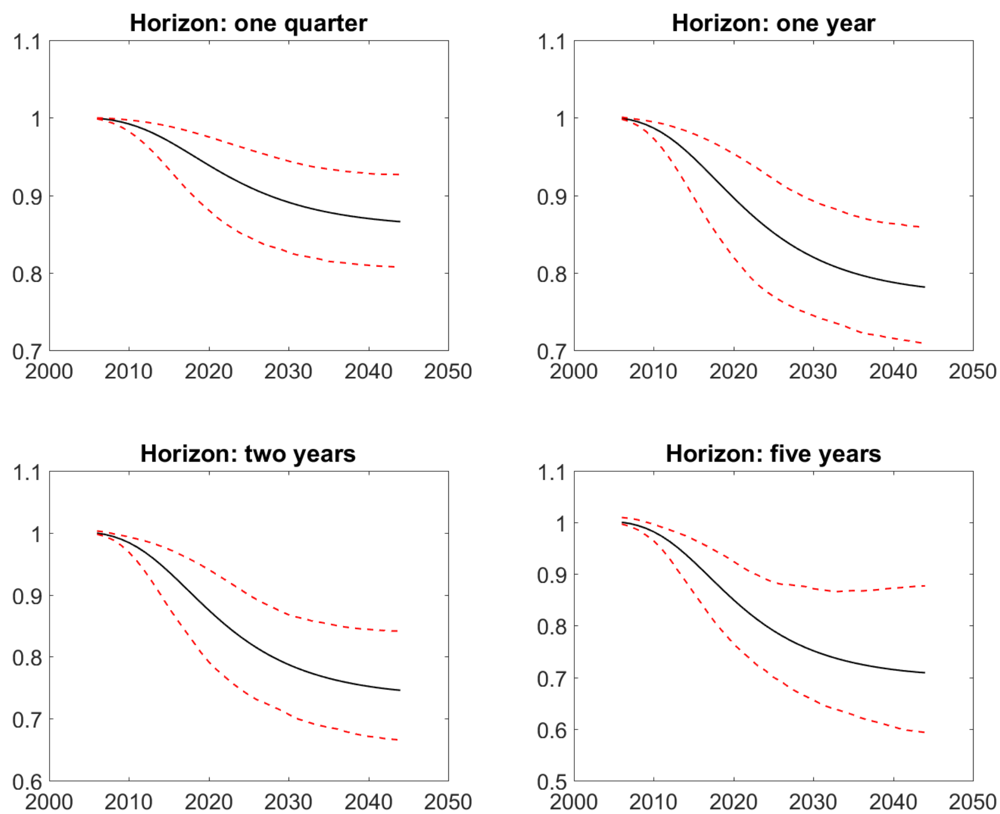
Note: The solid black line is the actual, the black dashed line is the mean of the posterior distribution for the expected transition path, and the red dashed lines are the 5th and 95th percentiles of the expected transition path.

Figure D2: Oil market and shale transition: Shale share = 25%
Mean, 5th and 95th percentiles of posterior distribution



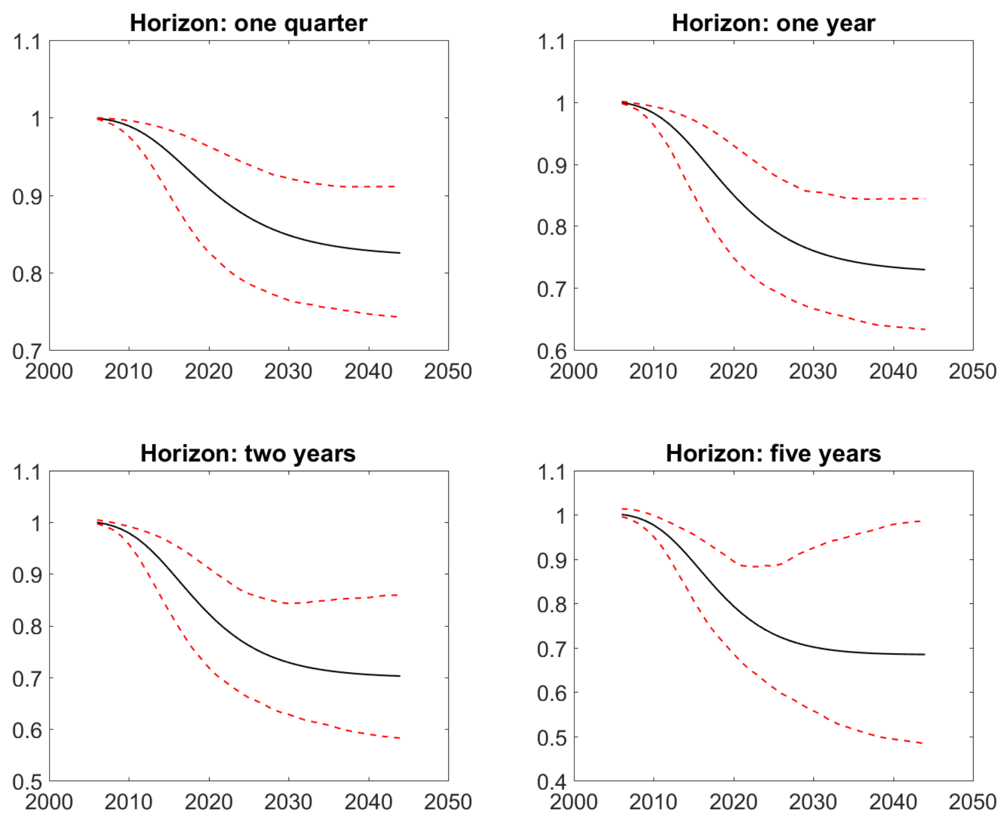
Note: The solid black line is the actual, the black dashed line is the mean of the posterior distribution for the expected transition path, and the red dashed lines are the 5th and 95th percentiles of the expected transition path.

Figure D3: Conditional variance of log real oil price along transition path relative to pre-shale variance: Shale share = 15%
Mean, 5th and 95th percentiles of posterior distribution



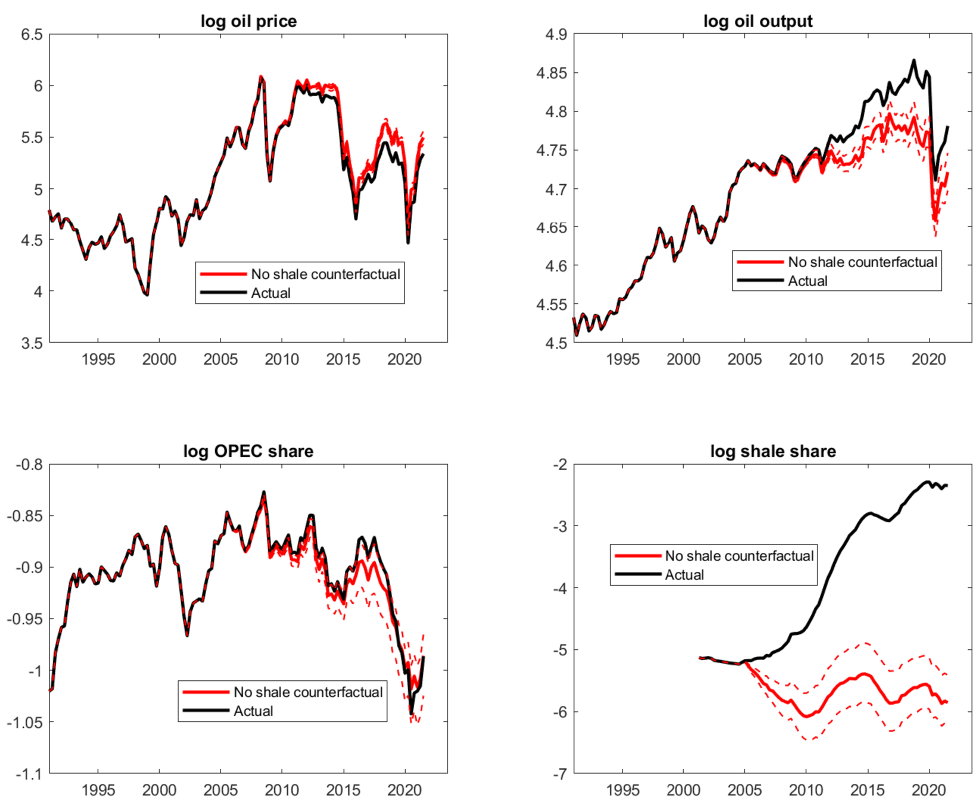
Note: The solid black line is the ratio of conditional forecast error variance at that time period relative to the conditional forecast error variance at the pre-shale steady state. The dashed red lines are the 5th and 95th percentiles of the posterior distribution.

Figure D4: Conditional variance of log real oil price along transition path relative to pre-shale variance: Shale share = 25%
Mean, 5th and 95th percentiles of posterior distribution



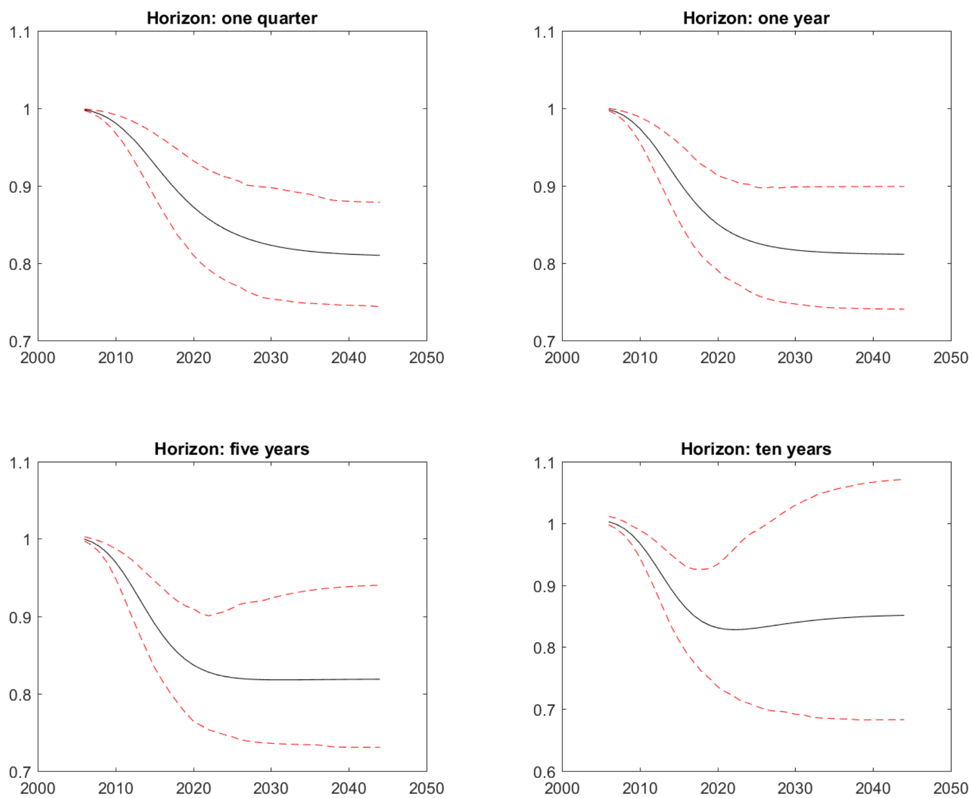
Note: The solid black line is the ratio of conditional forecast error variance at that time period relative to the conditional forecast error variance at the pre-shale steady state. The dashed red lines are the 5th and 95th percentiles of the posterior distribution.

Figure D5: Shale Revolution vs No Shale Revolution Counterfactual: Model with OPEC instead of OPEC Core



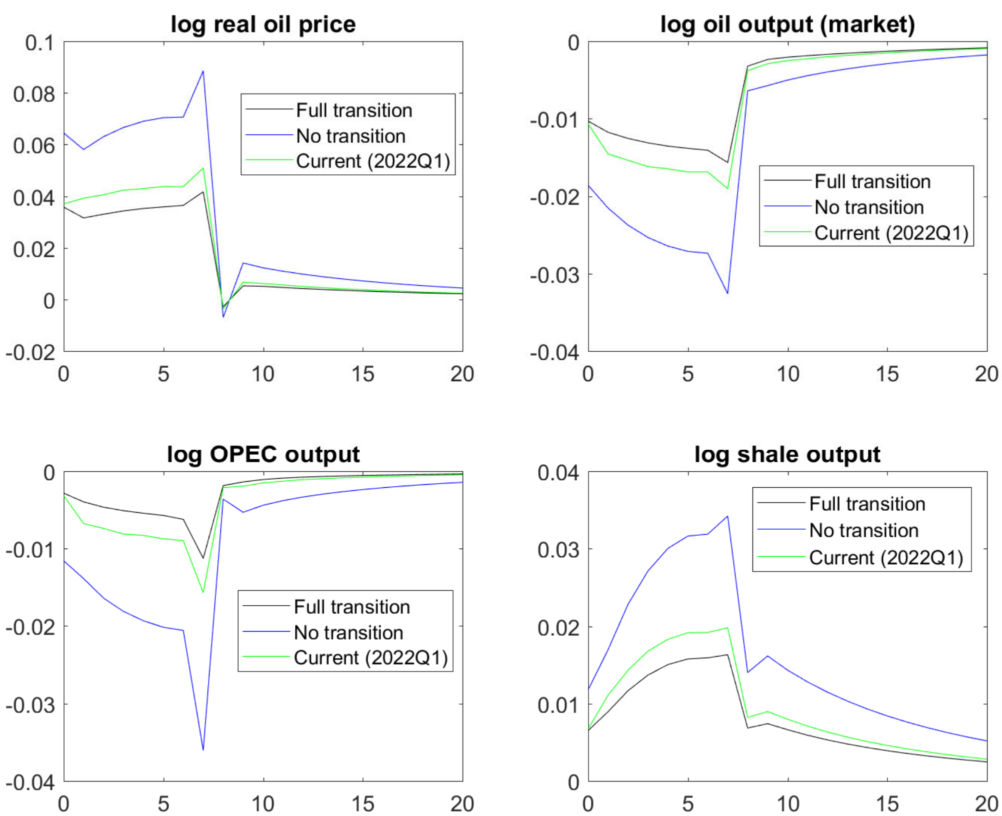
Note: The solid black line in the actual data. The solid red line is mean of the posterior distribution for the no-shale revolution counterfactual. The red dashed lines are the 5th and 95th percentiles of the posterior distribution.

Figure D6: Conditional variance of log real oil price along transition path relative to pre-shale variance: Model with OPEC instead of OPEC core
Mean, 5th and 95th percentiles of posterior distribution



Note: The solid black line is the ratio of conditional forecast error variance at that time period relative to the conditional forecast error variance at the pre-shale steady state. The dashed red lines are the 5th and 95th percentiles of the posterior distribution.

Figure D7: Response to shock in conventional fringe supply of 2.8%
Model with OPEC instead of OPEC Core



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