Extremal Equilibria in Repeated Oligopoly with Entry

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Collusion and Entry

- It is commonly accepted that collusion among firms is more likely to succeed when substantial entry barriers are present
  - Scherer (1980) and Harrington (1989)
  - DOJ and FTC 2023 Merger Guidelines: "entry barriers protect an incumbent from competition" and "increasing entry barriers generally entrench a dominant position..."
- Conversely, a promising strategy for policymakers to improve competition is to lower entry cost to stimulate new entrants (Starc and Wollmann 2022; Chassang and Ortner 2023)
- Systematic theoretical research on entry and collusion is rare (Harrington 1989; Stenbacka 1990)
- This paper provides a comprehensive analysis on the effects of entry on collusion by extending Abreu's optimal stick-and-carrot punishment to a repeated oligopoly with entry

This Paper

- We consider a canonical model of a repeated symmetric oligopoly with entry
  - Finite incumbent firms engage in Cournot competition over time
  - A large pool of entrants decides entry by incurring a fixed entry cost ($F > 0$)
  - Incumbent firms can deter entry with credible punishments (collective predation)
- Previous literature (Harrington 1989; Stenbacka 1990) has shown that "some" collusion can still be maintained for arbitrarily small F if the incumbent firms are patient enough (δ)
- We fully characterize the set of strongly symmetric equilibrium payoffs for a given (δ, F), following Abreu (1986)

★ Notably, the incumbent firms can counterintuitively use entry accommodation to **enhance** collusion!

  - Entry accommodation intensifies Abreu's optimal symmetric punishment, improving deterrence of quantity deviation by incumbent firms.
  - Technical difficulties: optimal symmetric punishment with entry, additional incentive constraints to accommodate and deter entry.

Results

- Optimal symmetric punishment with entry
  - if entry is used for punishment, how to "introduce" entrants? how many?
  - a simple **two-phase** structure suffices to construct optimal symmetric punishment with entry
- Linear demand
  1. Comparative statics of strongly symmetric equilibrium (SSE) payoffs of Abreu (1986) in number of firms (n): for fixed δ, the smallest SSE payoff $\underline{v}(n)$ decreases in n, and strictly so whenever $\underline{v}\,(n) > 0$
  2. The optimal punishment with entry either coincides with Abreu's optimal punishment (no entry accommodation) or improves Abreu's optimal punishment by accommodating **one or more entrants**
  3. Complete characterization of SSE payoff set for each (δ, F, n), which can expand strictly compared to that without using entry-accommodating punishment

Results

- Linear demand
  - if entry cost F is small, entry cannot be deterred (one or more entrants enter in equilibrium) but entry accommodation enables harsher punishments than Abreu (1986)
  - if entry cost F is intermediate, entry can be deterred and can be used to achieve harsher punishments than Abreu (1986)
  - if entry cost F is large, no entry on or off the equilibrium path with analysis the same as Abreu (1986)
- General demand
  - the same results obtain if the smallest SSE payoff $\underline{v}(n)$ strictly decreases in n when $\underline{v}(n) > 0$
  - if the demand function is smooth and log-concave, similar construction can be used to **improve** credible punishment
- Credible punishment can be further improved if we drop the requirement of symmetric equilibrium

Literature Review

- Theoretical analysis:
  - Abreu (1986)
  - Harrington (1989); Stenbacka (1990); Friedman and Thisse (1994)
  - Wiseman (2017)
- Empirical analysis:
  - Scott-Morton (1997): British shipping cartels
  - Harrington et al. (2018): German cement manufacturers
  - Asker and Hemphill (2020): Canadian sugar market
  - Starc and Wollmann (2022): US generic drug cartel
- More broadly on unintentional promotion of collusion:
  - Berheim (1984)
  - McCutcheon (1997)
  - Miller (2009), Spagnolo (2000), Chen and Harrington (2007)

Model with Linear Demand

- An infinitely repeated oligopoly game with $n \geq 2$ identical incumbent firms and δ. In each period,
  - firm i chooses quantity q_i of a homogeneous product at cost cq_i
  - market price determined as $p(z) = \alpha - z$ if $z = \sum_i q_i \in [0, \alpha]$; $p(z) = 0$ o/w
  - the past history of quantities is public information (perfect monitoring)

- There is a large pool of potential entrants
  - each entrant has the same production technology as incumbent firms, but has to pay entry cost $F > 0$
  - entry decisions are publicly observable at the beginning of each period
  - entry decisions are 'irreversible'

- Focus on strongly symmetric equilibrium (SSE) where after each history, the same quantity is chosen by all *active* (incumbent and newly entered) firms

Abreu's Optimal Symmetric Punishment

- A firm's one-shot payoff from quantity q :

$$\mu\left(q, n\right) = q \max\left\{p\left(nq\right), 0\right\} - cq$$

with $q^m\left(n\right)$ the unique monopoly quantity, $q^N\left(n\right)$ the unique NE quantity

- A firm's best one-shot deviation payoff given q chosen by the other firms:

$$\mu^D\left(q, n\right) = \max_{q' \in [0,\alpha]} q' \max\left\{p\left(q' + \left(n-1\right)q\right), 0\right\} - cq'$$

- The most collusive SSE quantity enforced by $\underline{v}(n)$, the minimum SSE payoff:

$$\overline{q}\left(n\right) := \arg\max_q \left\{\mu\left(q, n\right) \text{ s.t. } \mu\left(q, n\right) \geq \left(1-\delta\right)\mu^D\left(q, n\right) + \delta \underline{v}\left(n\right)\right\}$$

- An optimal **stick-and-carrot** punishment is a symmetric $\sigma\left(\widehat{q}\left(n\right), \overline{q}\left(n\right)\right)$

$$\left(1-\delta\right)\mu\left(\widehat{q}\left(n\right), n\right) + \delta\mu\left(\overline{q}\left(n\right), n\right) = \underline{v}\left(n\right)$$

$\sigma\left(\widehat{q}\left(n\right), \overline{q}\left(n\right)\right)$ specifies one period of $\widehat{q}\left(n\right)$ followed by repeated play of $\overline{q}\left(n\right)$, with deviation prompting the prescription to be repeated

Simple Characterization and Comparative Statics

Lemma (Optimal Symmetric Punishment without Entry)

Suppose $n \geq \frac{\alpha}{c}$. In the infinitely repeated oligopoly game **without entry**,

1. $\sigma(\widehat{q}(n), \overline{q}(n))$ *in the repeated oligopoly game satisfies*

$$\mu^D(\widehat{q}(n), n) = (1-\delta)\mu(\widehat{q}(n), n) + \delta\mu(\overline{q}(n), n) = \underline{v}(n) \text{ and}$$
$$\mu^D(\overline{q}(n), n) - \mu(\overline{q}(n), n) = \delta[\mu(\overline{q}(n), n) - \mu(\widehat{q}(n), n)] \text{ if } \overline{q}(n) > q^m(n)$$
$$\mu^D(\overline{q}(n), n) - \mu(\overline{q}(n), n) < \delta[\mu(\overline{q}(n), n) - \mu(\widehat{q}(n), n)] \text{ if } \overline{q}(n) = q^m(n).$$

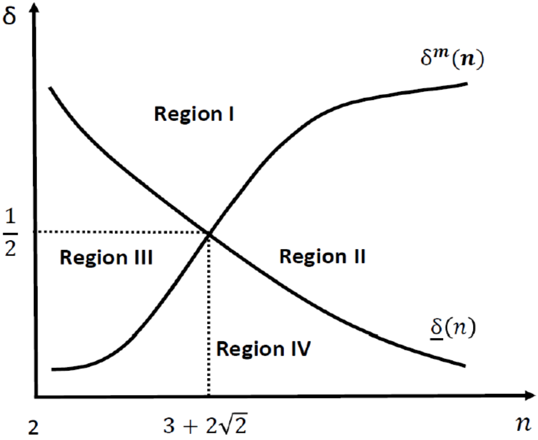
2. *any SSE payoff in* $[\underline{v}(n), \overline{v}(n)]$ *with* $\overline{v}(n) = \mu(\overline{q}(n), n)$ *is obtained by* $(q^*, \overline{q}(n))$ *s.t.* q^* *is in the convex set* $B := \{q : (1-\delta)[\mu^D(q, n) - \mu(q, n)] \leq \delta[\overline{v}(n) - \underline{v}(n)]\}$.

3. $\overline{q}(n)$ *and* $\underline{v}(n)$ *decrease in* δ; $\widehat{q}(n)$ *strictly increases in* $\delta \Longrightarrow$ *unique cutoffs* $\delta^m(n)$, $\underline{\delta}(n)$ *with "*$\underline{v}(n) = 0$ **iff** $\delta \geq \underline{\delta}(n)$*" and "*$\overline{q}(n) = q^m(n)$ **iff** $\delta \geq \delta^m(n)$*."*

4. $\underline{\delta}(n)$ **strictly decreases and** $\delta^m(n)$ **strictly increases in** n; $\overline{v}(n)$ **strictly decreases in** n **and** $\underline{v}(n)$ **strictly decreases in** n **for** $\underline{v}(n) > 0$.

A 'Complete' Picture of SSE without Entry

- Divide the (δ, n) space into four Regions
  - Region I: $\delta > \max\{\underline{\delta}(n), \delta^m(n)\}$ so that $\overline{q}(n) = q^m(n)$, $\underline{v}(n) = 0$
  - Region II: $\underline{\delta}(n) < \delta < \delta^m(n)$ so that $\overline{q}(n) > q^m(n)$, $\underline{v}(n) = 0$
  - Region III: $\delta^m(n) < \delta < \underline{\delta}(n)$ so that $\overline{q}(n) = q^m(n)$, $\underline{v}(n) > 0$
  - Region IV: $\delta < \min\{\underline{\delta}(n), \delta^m(n)\}$ so that $\overline{q}(n) > q^m(n)$, $\underline{v}(n) > 0$



Optimal Symmetric Punishment with Entry

- Denote **a two-phase path with entry** in a subgame with n_0 active firms as $(n_2, q_2(n_2); n_1, q_1(n_1); n_0)$ where $n_1 \geq n_0$ and $n_2 \geq n_1$
  - entrants $n_0 + 1, ..., n_1$ enter in period 1 to produce $q_1(n_1)$
  - entrants $n_1 + 1, ..., n_2$ enter in period 2 to produce $q_2(n_2)$ in periods 2, 3,....

Proposition (Sufficiency of Two-Phase Paths with Entry)

In the repeated oligopoly with a general demand function, for any strongly symmetric equilibrium where entry occurs on or off the equilibrium path, there exists a payoff equivalent two-phase path with entry which is also subgame perfect.

- Reducing an infinite-dimensional problem to a two-dimensional problem
- "Minimal" characterization, analogous to Abreu's optimal punishment
  - while the most collusive payoff is attainable via stationary paths, the most severe symmetric punishment has to use non-stationary paths
  - the simplest non-stationary paths are two-phase paths

Harsher Punishments

- Surprisingly, optimal symmetric punishment with entry can be used by incumbent firms to improve collusion with an intermediate entry barrier F.

Proposition (Improving Collusion via Entry)

If $(1-\delta)F \in (\underline{v}(n+1), \min\{\underline{v}(n), \overline{v}(n+1)\})$, *then the lowest symmetric equilibrium payoff in a subgame with n incumbents is exactly* $(1-\delta)F$, *which can be reached by a two-phase path with entry* $(n+1, \overline{q}(n+1); n+1, q^*(n+1); n)$ *for some* $q^*(n+1) \in [\overline{q}(n+1), \widehat{q}(n+1))$ *such that entry only occurs in the first period in equilibrium.*

- Intuition
  - as $\underline{v}(n)$ strictly decreases in n when $\underline{v}(n) > 0$, entry accommodation (more firms) leads to harsher punishments
  - entry accommodation however requires (1) providing incentives for an entrant to enter and (2) discouraging excessive entry
- Proof
  - Existence of an SSE with payoff $(1-\delta)F$
  - No SSE with lower payoff than $(1-\delta)F$ (two-phase paths with entry)

Harsher Punishments: non-uniqueness

- The optimal symmetric punishment with entry may not be unique

Proposition (Non-Uniqueness)

Suppose $(1-\delta)\,F \in (\underline{v}(n+1), \min\{\underline{v}(n), \overline{v}(n+\ell)\})$ *for* $\ell \geq 1$. *The lowest symmetric equilibrium payoff in a subgame with* n *incumbents is exactly* $(1-\delta)\,F$, *which is obtained by a two-phase path with entry* $(n+k, \overline{q}\,(n+k)\,; n+k, q^*\,(n+k)\,; n)$ *for some* $q^*\,(n+k) \in [\overline{q}\,(n+k)\,, \widehat{q}\,(n+k))$, k *entrants enter in the first period in equilibrium,* $k \leq \ell$.

- Hence, depending on parameters, we can accommodate $1, 2, \ldots, k$ entrants to construct multiple optimal symmetric punishments with entry
- But accommodating more entrants does not lead to harsher punishments
  - as $\overline{v}(n+1) \geq \overline{v}(n+\ell)$ for $\ell \geq 1$, accommodating exactly one entrant is feasible
  - the lowest SSE payoff is $(1-\delta)\,F$, regardless of $k \in \{1, \ldots, \ell\}$

Equilibrium Payoff Implications

- The harsher punishments from entry can expand the set of SSE payoffs

1. Entry accommodation is used **purely** as a (credible) threat
   - Improved carrot: $\overline{q}^E(n)$

$$\overline{q}^E(n) := \arg\max_q \mu(q,n) \text{ s.t. } (1-\delta)\mu^D(q,n) + \delta(1-\delta)F \leq \mu(q,n)$$

   - Improved stick: $\widehat{q}^E(n)$

$$\widehat{q}^E(n) := \arg\max_q \mu(q,n) \text{ s.t. } \begin{array}{c} (1-\delta)\mu^D(q,n) + \delta(1-\delta)F \\ \leq (1-\delta)\mu(q,n) + \delta\mu\left(\overline{q}^E(n), n\right) \end{array}$$

   - Improved extreme SSE payoffs

$$\begin{aligned} \overline{v}^E(n) &:= \mu\left(\overline{q}^E(n), n\right) \\ \underline{v}^E(n) &:= (1-\delta)\mu\left(\widehat{q}^E(n), n\right) + \delta\mu\left(\overline{q}^E(n), n\right) \end{aligned}$$

2. Entry accommodation can be credible, hence expanding SSE payoff **by itself**

$$\text{“additional” SSE payoff set: } \left[(1-\delta)F, \overline{v}(n+1)\right]$$

Complete SSE Characterization

- Varying the size of the entry barrier F enables us to obtain a full characterization of SSE payoffs when entry is present

Proposition (Full Characterization of SSE Payoffs)

In the repeated linear oligopoly game with entry barrier F and n incumbent firms,

1. *if* $(1-\delta)F \geq \underline{v}(n)$ *or* $(1-\delta)F > \overline{v}(n+1)$, *the SSE payoff set is* $[\underline{v}(n), \overline{v}(n)]$
2. *if* $(1-\delta)F \in (\underline{v}(n+1), \min\{\underline{v}(n), \overline{v}(n+1)\})$, *the SSE payoff set is* $[(1-\delta)F, \overline{v}(n+1)] \cup \left[\underline{v}^E(n), \overline{v}^E(n)\right]$
3. *if* $(1-\delta)F \leq \underline{v}(n+1)$, *then there is* $k > n$ *with* $(1-\delta)F \in (\underline{v}(k+1), \underline{v}(k)]$ *and the SSE payoff set is either* $[\underline{v}(k), \overline{v}(k)]$ *or* $[(1-\delta)F, \overline{v}(k+1)] \cup \left[\underline{v}^E(k), \overline{v}^E(k)\right]$

Complete SSE Characterization

- Depending of the size of F, excessive entry may or may not occur in the equilibrium

1. when F is large (Case 1), no entry on or off the equilibrium path
   - set of SSE payoffs is the **same** as Abreu (1986)
2. when F is intermediate (Case 2), entry can be used as a credible threat
   - set of SSE payoffs is **strictly larger** than Abreu (1986)
3. when F is small (Case 3), excessive entry inevitable but still can be used as a credible threat
   - set of SSE payoffs is "**strictly larger**" than Abreu (1986)

- The SSE payoff set here can be **non-convex**, unlike in Abreu (1986), as optimal punishment with entry creates discontinuity in the SSE payoff set
- Important takeaway:
  - **lowering entry cost F may facilitate or promote collusion!**

An Explicit Example

- $p(z) = \alpha - z$, $n = 3$, $\alpha = 2$, $c = 1$, $3c \geq \alpha$ (regularity), $\delta^m(3) = 1/3$, denote Abreu's SSE payoff set as $[\underline{v}, \overline{v}]$ and the set of SSE payoffs in our setting as $[\underline{u}, \overline{u}]$
- $\delta = 1/3$:
  1. Abreu (1986): $[\underline{v}(3), \overline{v}(3)] = [1/36, 1/12] \approx [0.028, 0.083]$, $[\underline{v}(4), \overline{v}(4)] = \left[\frac{169}{15625}, \frac{969}{15625}\right] \approx [0.0108, 0.062]$
  2. $(1-\delta)F = 0.03$, then $[\underline{u}, \overline{u}] = [\underline{v}(3), \overline{v}(3)] \approx [0.028, 0.083]$
  3. $(1-\delta)F = 0.02$, then $[\underline{u}, \overline{u}] = [(1-\delta)F, \overline{v}(n+1)] \cup [\underline{v}^E(n), \overline{v}^E(n)] = [(1-\delta)F, \overline{v}(n)] \approx [0.02, 0.083]$
- $\delta = 0.05$:
  1. Abreu (1986): $[\underline{v}(3), \overline{v}(3)] \approx [0.056, 0.068]$, $[\underline{v}(4), \overline{v}(4)] \approx [0.034, 0.045]$
  2. $(1-\delta)F = 0.044$, then $[\underline{u}, \overline{u}] = [0.044, 0.045] \cup [0.052, 0.07]$
  3. $(1-\delta)F = 0.03$, then entry **cannot** be deterred and $[\underline{u}, \overline{u}] = [(1-\delta)F, \overline{v}(4)] \approx [0.03, 0.045]$

General Demand Functions

- Results for the linear demand case generalize without change to the general-demand setting with
  - the extreme SSE payoffs in Abreu (1986) $\underline{v}(n)$ and $\overline{v}(n)$ are strictly decreasing in n
  - moderate entry barrier F
- $\underline{v}(n)$ and $\overline{v}(n)$ are however endogenous and finding necessary and sufficient conditions on fundamental parameters is difficult...
- Partial results:
  1. for a log-concave demand $p(z)$, if δ is small and moderate F near $\mu\left(q^N(n+1), n+1\right)$, then SSE payoffs can be lowered to $(1-\delta)F < \underline{v}(n+1)$
  2. for a concave demand $p(z)$, together with a technical assumption, SSE payoffs can be lowered to $(1-\delta)F < \underline{v}(n+1)$ for broader scenarios, i.e., $\delta \in [\underline{\delta}(n+1), \underline{\delta}(n))$ and small F
- The lowest SSE payoff is however unknown

Asymmetric Punishment

- If asymmetric punishments are feasible, then the two-phase (stick-and-carrot) punishment is insufficient when $\underline{v}(n) > 0$ (Abreu 1986)
  - the most severe asymmetric equilibrium may exhibit a 'flip-flop' pattern, i.e., a player with a high stage payoff always expects a lower continuation payoff
  - non-stationary strategies play a larger role in asymmetric equilibria
- We show that asymmetric strategies can indeed lead to harsher punishments than the optimal symmetric punishment with entry
- Consider the linear demand setting
  - the lowest SSE payoff $\underline{v}(n) = (1-\delta) F$
  - allow 'limited asymmetry' such that the active (incumbent and newly entered) firms' outputs can be different **only** in the first period of the optimal punishment with entry
  - it is possible to raise each entrant's payoff above $(1-\delta) F$ and lower the incumbents' equilibrium payoff to be 0
  - the limited asymmetric punishment can achieve **the most severe punishment** of $\underline{v}(n) = 0$

Takeaways

- We analyze extremal strongly symmetric equilibria (SSE) in a repeated oligopoly game with entry threats:
  1. comparative statics in n for Abreu (1986) with no entry, providing a more complete picture of SSE
  2. sufficient to focus on two-phase punishments with entry in analyzing SSE (with entry on or off the equilibrium path)
  3. entry accommodation can be strategically used to expand the set of SSE payoffs, facilitating collusion
- Implications for antitrust policies when regulating entry barriers in an industry

The End

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Thank You!