

Extremal Equilibria in Repeated Oligopoly with Entry

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Collusion and Entry

- It is commonly accepted that collusion among firms is more likely to succeed when substantial entry barriers are present
 - Scherer (1980) and Harrington (1989)
 - DOJ and FTC 2023 Merger Guidelines: “entry barriers protect an incumbent from competition” and “increasing entry barriers generally entrench a dominant position...”
- Conversely, a promising strategy for policymakers to improve competition is to lower entry cost to stimulate new entrants (Starc and Wollmann 2022; Chassang and Ortner 2023)
- Systematic theoretical research on entry and collusion is rare (Harrington 1989; Stenbacka 1990)
- This paper provides a comprehensive analysis on the effects of entry on collusion in a repeated oligopoly

- We consider a canonical model of a repeated symmetric oligopoly with entry
 - Finite (n) incumbent firms engage in Cournot competition over time
 - A large pool of entrants decides entry by incurring a fixed entry cost ($F > 0$)
 - Incumbent firms can coordinate to deter entry
 - Previous literature (Harrington 1989; Stenbacka 1990) has shown that “some” collusion can still be maintained for arbitrarily small F if the incumbent firms are patient enough (δ)
 - We generalize Abreu (1986) and fully characterize the set of strongly symmetric equilibrium payoffs for a given (n, δ, F)
- ★ Notably, the incumbent firms can counterintuitively use entry accommodation to **enhance** collusion!
- Entry accommodation intensifies Abreu’s optimal symmetric punishment
 - Difficulties: optimal symmetric punishment with entry, additional incentive constraints to accommodate and deter entry

- A simple **two-phase** structure suffices to construct optimal symmetric punishment with entry
- Linear demand
 - 1 Comparative statics of strongly symmetric equilibrium (SSE) payoffs of Abreu (1986) in number of firms (n)
 - 2 The optimal punishment with entry either coincides with Abreu's optimal punishment (no entry accommodation) or improves Abreu's optimal punishment by accommodating exactly **one entrant**
 - 3 Complete characterization of SSE payoff set for each (δ, F, n) , which can expand strictly compared to that in Abreu (1986)
- General demand

Similar construction can be used to improve credible punishment if the demand function is smooth and log-concave
- Credible punishment can be further improved if we drop the requirement of symmetric equilibrium

- Theoretical analysis:
 - Abreu (1986)
 - Harrington (1989); Stenbacka (1990); Friedman and Thisse (1994)
 - Wiseman (2017)
- Empirical analysis:
 - Scott-Morton (1997): British shipping cartels
 - Harrington et al. (2018): German cement manufacturers
 - Asker and Hemphill (2020): Canadian sugar market
 - Starc and Wollmann (2022): US generic drug cartel
- More broadly on unintentional promotion of collusion:
 - Berheim (1984)
 - McCutcheon (1997)
 - Miller (2009), Spagnolo (2000), Chen and Harrington (2007)

Model (Linear Demand)

- An infinitely repeated oligopoly game with $n \geq 2$ identical incumbent firms and δ . In each period,
 - firm i chooses quantity q_i of a homogeneous product at cost cq_i
 - market price $p(z) = \alpha - z$ if $z = \sum_i q_i \in [0, \alpha]$; $p(z) = 0$ otherwise
 - the past history of play is public information (perfect monitoring)
- There is a large pool of potential entrants
 - same production technology, but has to pay entry cost $F > 0$ to enter
 - entry decisions are publicly observable at the beginning of each period
 - entry decisions are 'irreversible'
- Focus on strongly symmetric equilibrium (SSE) where after each history, the same quantity is chosen by all *active* (incumbent and newly entered) firms

Abreu's Optimal Symmetric Punishment

- A firm's one-shot payoff from quantity q :

$$\mu(q, n) = q \max\{p(nq), 0\} - cq$$

with $q^m(n)$ the unique monopoly quantity, $q^N(n)$ the unique NE quantity

- A firm's best one-shot deviation payoff given q chosen by the other firms:

$$\mu^D(q, n) = \max_{q' \in [0, \alpha]} q' \max\{p(q' + (n-1)q), 0\} - cq'$$

- The most collusive SSE quantity enforced by $\underline{v}(n)$, the minimum SSE payoff:

$$\bar{q}(n) := \arg \max_q \left\{ \mu(q, n) \text{ s.t. } \mu(q, n) \geq (1 - \delta) \mu^D(q, n) + \delta \underline{v}(n) \right\}$$

- An optimal **stick-and-carrot** punishment is a symmetric $\sigma(\hat{q}(n), \bar{q}(n))$

$$(1 - \delta) \mu(\hat{q}(n), n) + \delta \mu(\bar{q}(n), n) = \underline{v}(n)$$

$\sigma(\hat{q}(n), \bar{q}(n))$ specifies one period of $\hat{q}(n)$ followed by repeated play of $\bar{q}(n)$, with deviation prompting the prescription to be repeated

Simple Characterization and Comparative Statics

Lemma (Optimal Symmetric Punishment without Entry)

Suppose $n \geq \frac{\alpha}{c}$. In the infinitely repeated oligopoly game **without entry**,

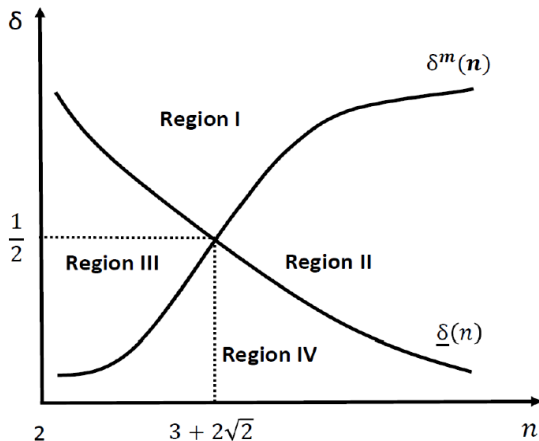
- ① $\sigma(\hat{q}(n), \bar{q}(n))$ in the repeated oligopoly game satisfies

$$\begin{aligned}\mu^D(\hat{q}(n), n) &= (1 - \delta)\mu(\hat{q}(n), n) + \delta\mu(\bar{q}(n), n) = \underline{v}(n) \text{ and} \\ \mu^D(\bar{q}(n), n) - \mu(\bar{q}(n), n) &= \delta[\mu(\bar{q}(n), n) - \mu(\hat{q}(n), n)] \text{ if } \bar{q}(n) > q^m(n) \\ \mu^D(\bar{q}(n), n) - \mu(\bar{q}(n), n) &< \delta[\mu(\bar{q}(n), n) - \mu(\hat{q}(n), n)] \text{ if } \bar{q}(n) = q^m(n).\end{aligned}$$

- ② any SSE payoff in $[\underline{v}(n), \bar{v}(n)]$ with $\bar{v}(n) = \mu(\bar{q}(n), n)$ is obtained by $(q^*, \bar{q}(n))$ s.t. q^* is in the convex set $B := \{q : (1 - \delta)[\mu^D(q, n) - \mu(q, n)] \leq \delta[\bar{v}(n) - \underline{v}(n)]\}$.
- ③ $\bar{q}(n)$ and $\underline{v}(n)$ decrease in δ ; $\hat{q}(n)$ strictly increases in $\delta \implies$ cutoffs $\delta^m(n)$, $\underline{\delta}(n)$ with " $\underline{v}(n) = 0$ iff $\delta \geq \underline{\delta}(n)$ " and " $\bar{q}(n) = q^m(n)$ iff $\delta \geq \delta^m(n)$."
- ④ $\underline{\delta}(n)$ strictly decreases and $\delta^m(n)$ strictly increases in n ; $\bar{v}(n)$ strictly decreases in n and $\underline{v}(n)$ strictly decreases in n for $\underline{v}(n) > 0$.

A 'Complete' Picture of SSE without Entry

- Divide the (δ, n) space into four Regions
 - Region I: $\delta > \max\{\underline{\delta}(n), \delta^m(n)\}$ so that $\bar{q}(n) = q^m(n)$, $\underline{v}(n) = 0$
 - Region II: $\underline{\delta}(n) < \delta < \delta^m(n)$ so that $\bar{q}(n) > q^m(n)$, $\underline{v}(n) = 0$
 - Region III: $\delta^m(n) < \delta < \underline{\delta}(n)$ so that $\bar{q}(n) = q^m(n)$, $\underline{v}(n) > 0$
 - Region IV: $\delta < \min\{\underline{\delta}(n), \delta^m(n)\}$ so that $\bar{q}(n) > q^m(n)$, $\underline{v}(n) > 0$



Optimal Symmetric Punishment with Entry

- Denote a **two-phase path with entry** in a subgame with n_0 active firms as $(n_2, q_2(n_2); n_1, q_1(n_1); n_0)$ where $n_1 \geq n_0$ and $n_2 \geq n_1$
 - entrants $n_0 + 1, \dots, n_1$ enter in period 1 to produce $q_1(n_1)$
 - entrants $n_1 + 1, \dots, n_2$ enter in period 2 to produce $q_2(n_2)$ in periods 2, 3,

Proposition (Sufficiency of Two-Phase Paths with Entry)

In the repeated oligopoly with a general demand function, for any strongly symmetric equilibrium where entry occurs on or off the equilibrium path, there exists a payoff equivalent two-phase path with entry which is also subgame perfect.

- Reducing an infinite-dimensional problem to a finite-dimensional problem
- “Minimal” characterization, analogous to Abreu’s optimal punishment
 - while the most collusive payoff is attainable via stationary paths, the most severe symmetric punishment has to use non-stationary paths
 - the simplest non-stationary paths are two-phase paths

Harsher Punishments

- Surprisingly, optimal symmetric punishment with entry can be used by incumbent firms to improve collusion with an intermediate entry barrier F .

Proposition (Improving Collusion via Entry)

If $(1 - \delta) F \in (\underline{v}(n + 1), \min\{\underline{v}(n), \bar{v}(n + 1)\})$, then the lowest symmetric equilibrium payoff in a subgame with n incumbents is exactly $(1 - \delta) F$, which can be reached by a two-phase path with entry $(n + 1, \bar{q}(n + 1); n + 1, q^*(n + 1); n)$ for some $q^*(n + 1) \in [\bar{q}(n + 1), \hat{q}(n + 1))$ such that entry only occurs in the first period in equilibrium.

• Intuition

- as $\underline{v}(n)$ strictly decreases in n when $\underline{v}(n) > 0$, entry accommodation (more firms) leads to harsher punishments
- entry accommodation however requires (1) providing incentives for an entrant to enter and (2) discouraging excessive entry

• Proof

- Existence of an SSE with payoff $(1 - \delta) F$
- No SSE with lower payoff than $(1 - \delta) F$ (two-phase paths with entry)

Equilibrium Payoff Implications

- The harsher punishments from entry can expand the set of SSE payoffs
- ④ Entry accommodation is used **purely** as a (credible) threat

- Improved carrot: $\bar{q}^E(n)$

$$\bar{q}^E(n) := \arg \max_q \mu(q, n) \text{ s.t. } (1 - \delta) \mu^D(q, n) + \delta(1 - \delta) F \leq \mu(q, n)$$

- Improved stick: $\hat{q}^E(n)$

$$\hat{q}^E(n) := \arg \max_q \mu(q, n) \text{ s.t. } \begin{aligned} &(1 - \delta) \mu^D(q, n) + \delta(1 - \delta) F \\ &\leq (1 - \delta) \mu(q, n) + \delta \mu(\bar{q}^E(n), n) \end{aligned}$$

- Improved extreme SSE payoffs

$$\underline{v}^E(n) : = \mu(\bar{q}^E(n), n)$$

$$\underline{v}^E(n) : = (1 - \delta) \mu(\hat{q}^E(n), n) + \delta \mu(\bar{q}^E(n), n)$$

- ② Entry accommodation can be credible, hence expanding SSE payoff **by itself**

“additional” SSE payoff set: $[(1 - \delta) F, \bar{v}(n + 1)]$

Complete SSE Characterization

- Varying the size of the entry barrier F enables us to obtain a full characterization of SSE payoffs when entry is present

Proposition (Full Characterization of SSE Payoffs)

In the repeated linear oligopoly game with entry barrier F and n incumbent firms,

- 1 *if $(1 - \delta) F \geq \underline{v}(n)$ or $(1 - \delta) F > \bar{v}(n + 1)$, the SSE payoff set is $[\underline{v}(n), \bar{v}(n)]$*
- 2 *if $(1 - \delta) F \in (\underline{v}(n + 1), \min\{\underline{v}(n), \bar{v}(n + 1)\})$, the SSE payoff set is $[(1 - \delta) F, \bar{v}(n + 1)] \cup [\underline{v}^E(n), \bar{v}^E(n)]$*
- 3 *if $(1 - \delta) F \leq \underline{v}(n + 1)$, then there is $k > n$ with $(1 - \delta) F \in (\underline{v}(k + 1), \underline{v}(k))$ and the SSE payoff set is either $[\underline{v}(k), \bar{v}(k)]$ or $[(1 - \delta) F, \bar{v}(k + 1)] \cup [\underline{v}^E(k), \bar{v}^E(k)]$*

Complete SSE Characterization

- Depending of the size of F , excessive entry may or may not occur in the equilibrium
- ① when F is large (Case 1), no entry on or off the equilibrium path
 - set of SSE payoffs is the **same** as Abreu (1986)
- ② when F is intermediate (Case 2), entry can be used as a credible threat
 - set of SSE payoffs is **strictly larger** than Abreu (1986)
- ③ when F is small (Case 3), excessive entry inevitable but still can be used as a credible threat
 - set of SSE payoffs is “**strictly larger**” than Abreu (1986)
- The SSE payoff set here can be **non-convex**, unlike in Abreu (1986), as optimal punishment with entry creates discontinuity in the SSE payoff set
- Important takeaway:
lowering entry cost F may facilitate or promote collusion!

An Example

- $p(z) = \alpha - z$, $n = 3$, $\alpha = 2$, $c = 1$, $3c \geq \alpha$ (regularity), $\delta^m(3) = 1/3$, denote Abreu's SSE payoff set as $[\underline{v}, \bar{v}]$ and the set of SSE payoffs in our setting as $[\underline{u}, \bar{u}]$
- $\delta = 1/3$:
 - 1 Abreu (1986): $[\underline{v}(3), \bar{v}(3)] = [1/36, 1/12] \approx [0.028, 0.083]$,
 $[\underline{v}(4), \bar{v}(4)] = [\frac{169}{15625}, \frac{969}{15625}] \approx [0.0108, 0.062]$
 - 2 $(1 - \delta)F = 0.03$, then $[\underline{u}, \bar{u}] = [\underline{v}(3), \bar{v}(3)] \approx [0.028, 0.083]$
 - 3 $(1 - \delta)F = 0.02$, then $[\underline{u}, \bar{u}] = [(1 - \delta)F, \bar{v}(n + 1)] \cup [\underline{v}^E(n), \bar{v}^E(n)] = [(1 - \delta)F, \bar{v}(n)] \approx [0.02, 0.083]$
- $\delta = 0.05$:
 - 1 Abreu (1986): $[\underline{v}(3), \bar{v}(3)] \approx [0.056, 0.068]$, $[\underline{v}(4), \bar{v}(4)] \approx [0.034, 0.045]$
 - 2 $(1 - \delta)F = 0.044$, then $[\underline{u}, \bar{u}] = [0.044, 0.045] \cup [0.052, 0.07]$
 - 3 $(1 - \delta)F = 0.034$, then entry **cannot** be deterred and
 $[\underline{u}, \bar{u}] = [(1 - \delta)F, \bar{v}(4)] \approx [0.034, 0.045]$

- We analyze extremal strongly symmetric equilibria (SSE) in a repeated oligopoly game with entry threats:
 - ① comparative statics in n for Abreu (1986) with no entry, providing a more complete picture of SSE
 - ② sufficient to focus on two-phase punishments with entry in analyzing SSE (with entry on or off the equilibrium path)
 - ③ entry accommodation can be strategically used to expand the set of SSE payoffs, facilitating collusion
- Implications for antitrust policies when regulating entry barriers in an industry

The End

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Thank You!