Understanding Power Grid Network Vulnerability through the Stochastic Lens of Network Motif Evolution

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Abstract

Many modern systems and collections of components/devices can be represented as complex networks. These networks such as, for instance, the internet, power grids, and water supply chains, are expected to exhibit high reliability levels since failures of these systems can lead to catastrophic cascading events. As a result, enhancing our understanding of mechanisms behind functionality and reliability of such networks is the key toward ensuring security, sustainability, and resilience of most modern critical infrastructures. While there exists a broad variety of statistical methods for assessing reliability of networks, most existing techniques utilize only a single network topological metric. Focusing on power grid applications, in this paper we develop

- a new stochastic model approach based on multiple interdependent topological measures of complex networks.
- Gamma degradation model to evaluate dynamics of multiple network motifs as descriptors of the underlying network topology and its response to adverse events.
- a formal statistical inference for analysis of reliability and robustness levels of a single complex network as well as for assessing differences in reliability properties exhibited by two different networks.
- the proposed new methodology with extensive Monte Carlo simulation studies and illustrate the utility of the new approach in application to vulnerability analysis of European power grid networks under various targeted attacks.

Graph Representation of Power System

- Consider a graph $G=(V,E)$ as a model for an electric power system, with node set V and set of edges $E \subset V\times V$. Here $e_{uv} \in E$ represents an edge, i.e., flow in line/transformer between nodes u and v, and nodes represent, e.g., generators ($\mathcal{G}$), loads ($\mathcal{L}$), and buses ($\mathcal{B}$).

- These three assumptions are described in terms of alternative approaches for power system functionality study:
  - assume that G is *unweighted-directed* graph i.e., for all edges e_{uv} have a direction associated with them and all edges weights w_{uv} are equal.
  - assume that G is *unweighted-undirected* graph i.e., for all edges $e_{uv} \in E, w_{uv} \equiv w_{vu}=1$.
  - assume that G is *weighted-undirected* graph i.e., each edge $e_{uv} \in E$ has a weight w_{uv} and $w_{uv}=w_{vu}$.

Network Motif Analysis

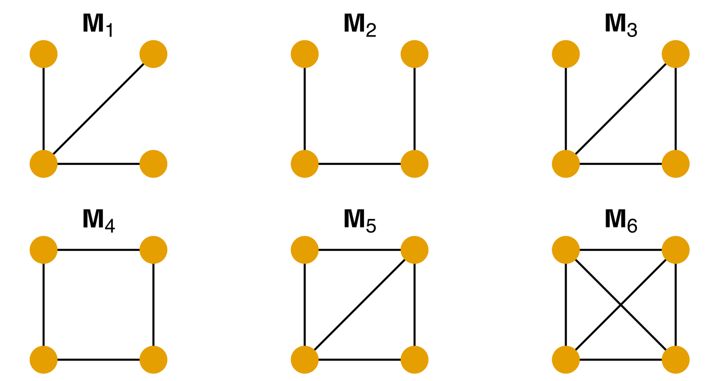


Figure 1: Undirected motifs on four vertices

- Let $G_\ell=(V_\ell,E_\ell)$ be a ℓ-node subgraph of G and a motif signature $f_G(G_\ell)$ be a number of occurrences of G_ℓ [1].
- Identify those sub-graphs have higher frequency in the graph G much more than in random graphs.
- Small sub-graphs found throughout a network capture its local structure.
- Motifs are intrinsically connected to the resilience of power system [2].

Gamma Degradation Model

- In network analysis based on motif, the process that the number of network motifs gradually decreases under random failures or intentional attacks can be considered as a degradation process.
- The commonly used degradation model is the gamma degradation model which describe the degradation process by a gamma process with stationary increments/decrements.
- By using remaining ℓ-node motifs under targeted attack the gamma degradation model learns both local and global structural properties of the network.
- We assume the decrements are independent random variables with shape parameters $\boldsymbol{\lambda}=(\lambda_{i,1},\cdots,\lambda_{i,\mathcal{J}})$ ($\mathcal{J}$ motifs in ℓ–node motifs), scale parameter β_i which is intended to capture the dependence among different motifs within the $Network_i$.

Dataset

To illustrate utility of the new methodology, we study the data from the Union for the Coordination of Transport of Electricity (UCTE) that includes power grid networks of 15 European countries. For illustrative purposes, we consider six European countries: France, Germany, Italy, Poland, Romania and Spain. The nodes in each network represent the various substations, and the edges represent the high voltage transmission lines that connect different substations. To measure the remaining functionality of power grid network, we focus on four types of 4-node motifs, $\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$ and $\mathbf{M_4}$, because of motifs $\mathbf{M_5}$ or $\mathbf{M_6}$ are not available in some of the power grid networks.

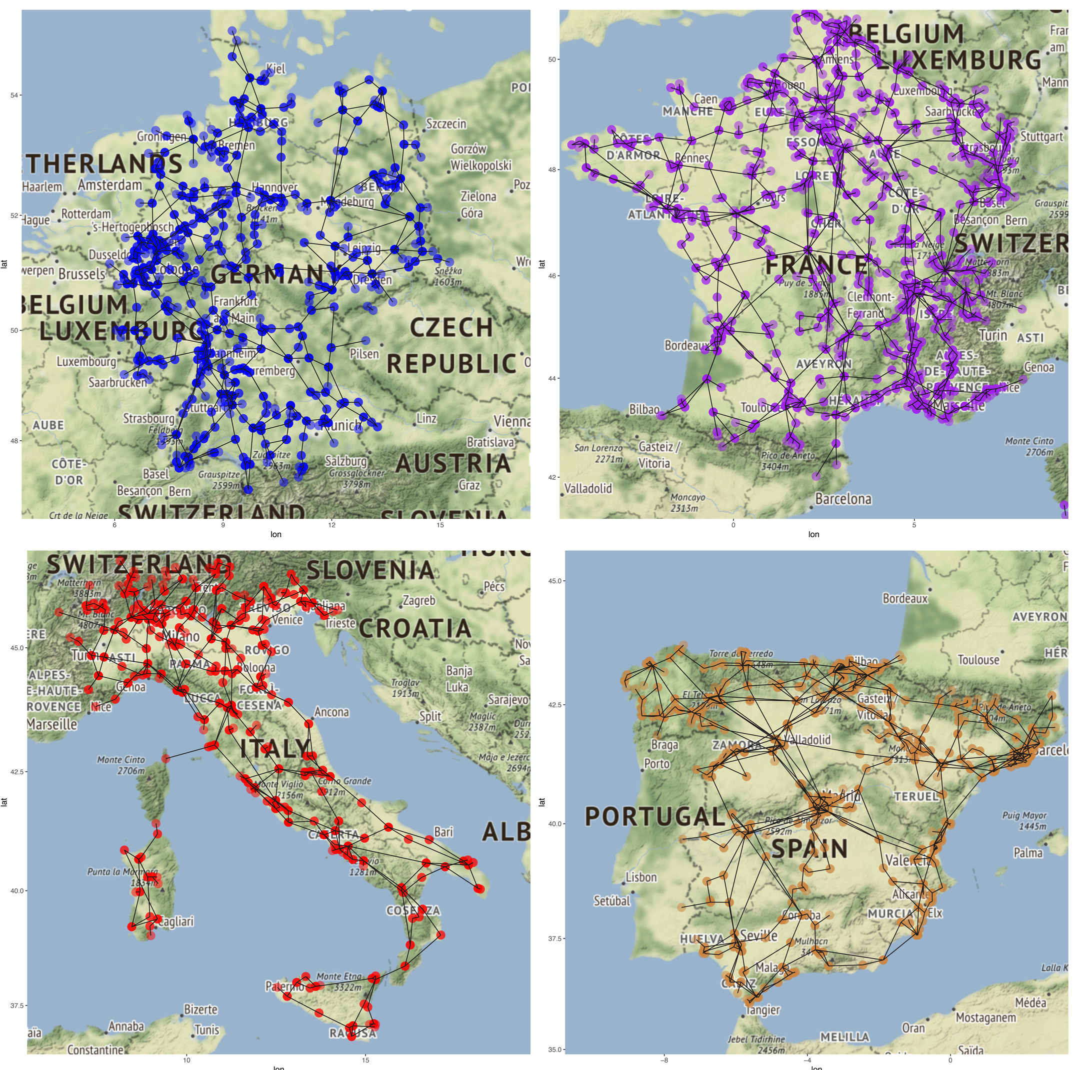


Figure 2: Maps representing the four European countries power grid networks (i.e., German, France, Italy and Spain), where nodes indicate generators, loads or buses

Method

⌘ **Data preprocessing:** (i) calculate the remaining numbers of the ℓ-node motifs at an observation point t_k, $t=1,\cdots,\mathcal{K}$. Let $x_{i,j,k}$ be the number of the i-th ℓ-node motif for power grid $Network_i$ at observation point t_k; (ii) assume $y_{i,j,k}=\log(x_{i,j,k}/x_{i,j,k+1})$ and $y_{i,j,k}$ are independent gamma random variable with shape parameter $\lambda_{i,j}>0$, and scale parameter $\beta_i>0$ denoted as $y_{i,j,k}\sim Ga(\lambda_{i,j},\beta_i)$.

⌘ **Model and parameter estimation:** feed the decrements $y_{i,j,k}$ into the log-likelihood function for the $Network_i$ as $\mathcal{L}_i(\Theta_i)=\prod_{j=1}^{\mathcal{J}}\prod_{k=1}^{\mathcal{K}}\frac{y_{i,j,k}^{\lambda_{i,j}}}{\Gamma(\lambda_{i,j})\beta_i^{\lambda_{i,j}}}\exp(\frac{-y_{i,j,k}}{\beta_i})$ and $\Theta_i=\left(\lambda_{i,1},\cdots,\lambda_{i,\mathcal{J}},\beta_i\right)^{\mathrm{T}}$ and obtain the unknown model parameters by maximizing $\mathcal{L}_i(\Theta_i)$.

⌘ **Hypothesis testing:** likelihood test with the gamma degradation model to assess whether two power grid networks are in the same resilience level:

$$\mathbf{H_0}: \mathcal{R}(Network_i)=\mathcal{R}(\mathrm{Network_{i'}}) \text{ versus } \mathbf{H_1}: \mathcal{R}(\mathrm{Network_i})\neq\mathcal{R}(\mathrm{Network_{i'}})$$

⌘ **Rank the resilience level:** calculate the mean degradation rate for each power grid network, namely, $\bar{\mathcal{A}}_i=\sum_{j=1}^{\mathcal{J}}\lambda_{i,j}\beta_i/\mathcal{J}$.

Decay Curves

Figure 3 illustrates the remaining (%) 4-node motifs ($\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$, and $\mathbf{M_4}$) of the six European power grids graphically under degree-centrality-based attacks. Note that the degradation rates of different 4-node motifs are different when the same fraction of nodes have been removed. For example, we find that the motif $\mathbf{M_3}$ in Poland (black curve) degrades faster than other countries, however, the motif $\mathbf{M_4}$ in Poland degrades slower than the other countries.

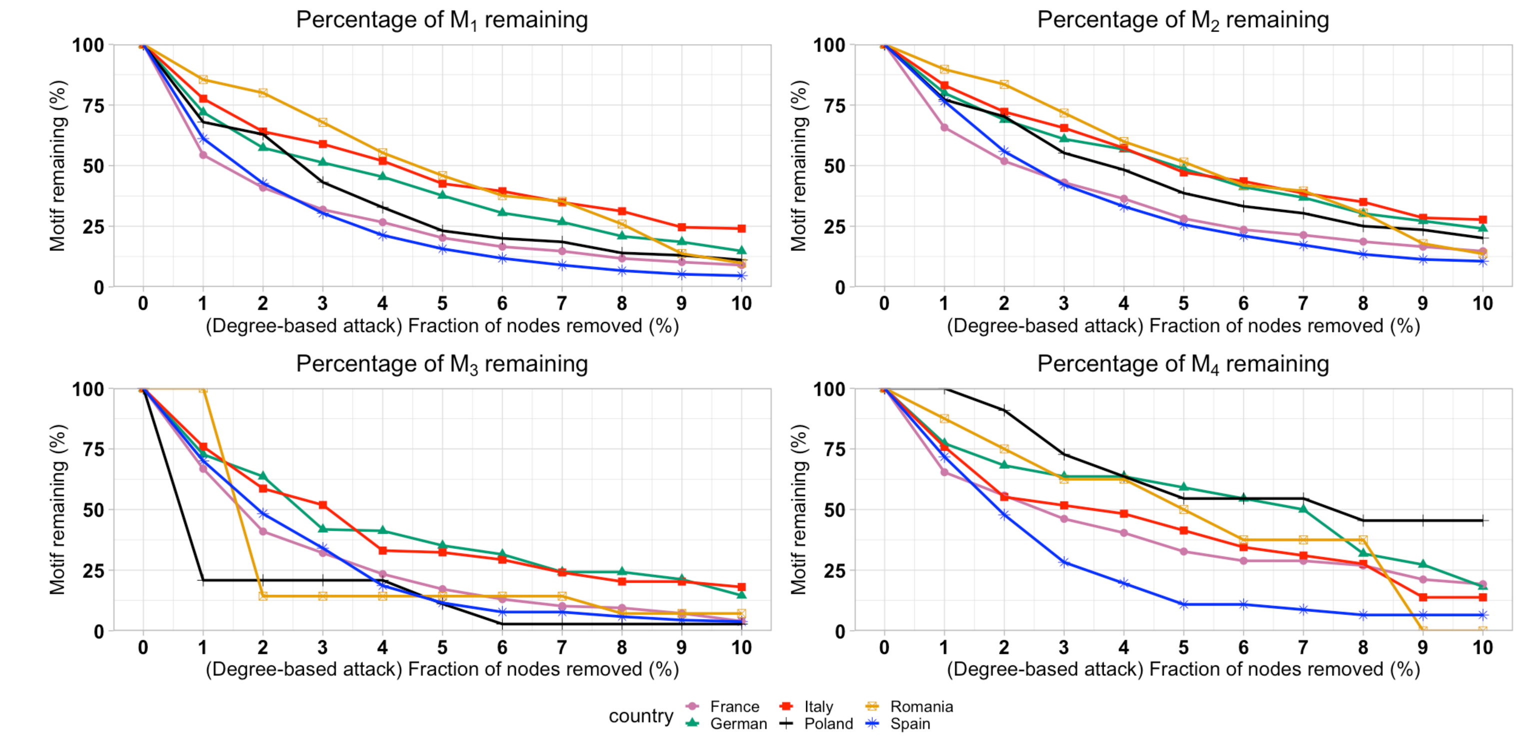
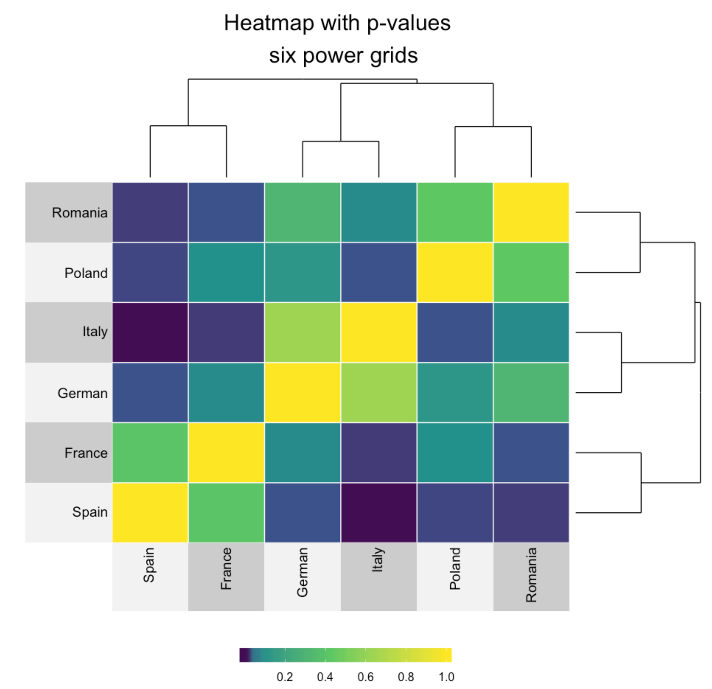


Figure 3: Degradation of motifs of six power grids under degree-based attack



Based on the heatmap, we can observe, for example, in contrast to other power grids, the robustness of the Italy power grid is similar to that of the Germany power grid; moreover, France power grid confers similar robustness to Spain power grid. To evaluate the rankings of the six power grids in terms of robustness, we can obtain two possible robustness rankings based on the heatmap:

- $[\text{Italy, Germany}] > [\text{Romania, Poland}] > [\text{France, Spain}]$

or

- $[\text{France, Spain}] > [\text{Romania, Poland}] > [\text{Italy, Germany}]$

Results

The mean degradation rate provides a systematic and quantitative way of ranking European power grid networks' resilience (see Table below).

	Degree-based attack					
Country	λ_1	λ_2	λ_3	λ_4	β	$\bar{\mathcal{A}}$
Germany	3.082	2.445	2.383	2.161	0.069	0.174
Italy	2.368	2.270	2.437	3.080	0.063	0.159
Poland	1.917	1.633	2.245	0.721	0.125	0.203
Romania	1.627	1.477	0.967	1.033	0.162	0.207
France	4.055	3.399	5.295	2.576	0.060	0.230
Spain	4.544	3.390	4.483	3.252	0.072	0.282

The smaller the $\bar{\mathcal{A}}$, the more robust the i-th network. Based on the values of $\bar{\mathcal{A}}$ in Table we can observe the average degradation rates of the motifs of Italy and Germany are much smaller than those of Romania, Poland, France and Spain, and the average of degradation rates of Romania and Poland are smaller than those of France and Spain.

Reference

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