Power System Event Location via DEIM

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Key Outcomes

Fast event location via DEIM: We propose the Discrete Empirical Interpolation Method (DEIM) index selection procedure for locating the source of power system events from Phasor Measurement Unit (PMU) data in real-time.

Numerical experiments using synthetically generated data from event simulations of the IEEE-68 bus test system illustrate that DEIM accurately locates the source of events.

Background and Motivation

Problem and motivation: Power system events (e.g., faults, load changes, line trips, etc.) happen irregularly and can lead to system wide oscillations. This motivates the development of fast, robust data-driven methods based on real-time synchrophasor measurements for detection, identification, and *location* of events.

Example: The 2003 Northeastern United States Blackout started out as a local fault that escalated to regional blackout due to lack of early intervention [1].

Low-Rank Matrix Factorizations of PMU Data

Phasor Measurement Units (PMUs): PMUs sample bus voltages, currents, etc., at each location up to 60 times per second in real-time.

Collected measurements reflect spatial-temporal variations in the current operating status of the network, and irregular deviations in the measured quantities reveal possible disturbances. We analyze PMU data in "spatial-temporal blocks" [2]:

$$\mathbf{Y} = \begin{bmatrix} | & | & & | \\ \mathbf{y}(t_1) & \mathbf{y}(t_2) & \cdots & \mathbf{y}(t_T) \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times T}.$$

- **Rows:** Correspond to measurements $y_i(t)$ at specific buses $i = 1, \ldots, n$.
- **Columns:** Correspond to measurements across all buses at a specific timestamp $t_i \geq 0$, $i = 1, 2, \ldots, T$.

Goal: Exploit the large amount of available PMU data to monitor and analyze the operating condition of the power system in real-time.

Challenge: Increased deployment of sensors plus high sampling rates lead to high-dimensional data sets that render real-time analysis (i.e., event monitoring) infeasible.

Solution: Use low-rank matrix factorizations to analyze the data in terms of a few dominant components [2]. Given $\mathbf{Y} \in \mathbb{R}^{n \times T}$, $n \ll T$, let $\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$ be the *singular value decomposition* of $\mathbf{Y}$. Then, the optimal rank$-k$ approximation $(k < n)$ is

$$\mathbf{Y} \approx \mathbf{Y}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{V}_k^T,$$

where $\mathbf{U}_k \in \mathbb{R}^{n \times k}$, $\boldsymbol{\Sigma}_k \in \mathbb{R}^{k \times k}$, and $\mathbf{V}_k \in \mathbb{R}^{T \times k}$.

Event Location via Leverage Scores

Note: The rows $(\mathbf{U}_k \boldsymbol{\Sigma}_k)_i$, $i = 1, \ldots, n$ contain the (truncated) coordinates of the measurements $y_i(t)$ in the basis of right singular vectors $\mathbf{V}$.

Leverage scores: Method for choosing "representative indices" of singular vectors.

Given: $\mathbf{Y}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{V}_k^T$, $\mathbf{U}_k \in \mathbb{R}^{n \times k}$, $k \ll n$. Define the *leverage score* of row i of $\mathbf{Y}_k$ as the vector 2-norm of the ith row of $\mathbf{U}_k \boldsymbol{\Sigma}_k$, that is:

$$\ell_i^k = \| (\mathbf{U}_k \boldsymbol{\Sigma}_k)(i, :) \|_2.$$

Idea (from [2]) **:** Sort buses in order of descending ℓ_i^k and locate the k buses most affected by the event by choosing the rows with the k highest leverage scores ℓ_i^k.

A large ℓ_i^k suggests possibly significant deviations at the bus where the measurement $y_i(t)$ was taken $\implies$ these can be used to locate the source of the event.

The DEIM Index Selection Algorithm

We propose an alternative methodology for locating power system events using the Discrete Empirical Interpolation Method (DEIM) [3] and its variants.

Main idea: The DEIM index selection procedure is a greedy algorithm that parses the dominant singular vectors $\mathbf{U}_k$ of a matrix $\mathbf{Y}$ in order to identify "representative" row indices $\mathbf{p} = \{p_1, \ldots, p_k\}$ of the data.

Algorithm 1 DEIM Index Selection Algorithm

input: $\{\mathbf{u}_i\}_{i=1}^k \subset \mathbb{R}^n$
output: $\mathbf{p} = \{p_i\}_{i=1}^k$
$p_1 = \arg\max |\mathbf{u}_1|$
$\mathbf{p} = \{p_1\}$, $\mathbf{P} = [\mathbf{e}_{p_1}]$, $\mathbf{U} = [\mathbf{u}_1]$
for $i = 2, \ldots k$
  solve $(\mathbf{P}^T\mathbf{U})\mathbf{c} = \mathbf{P}\mathbf{u}_i$ for $\mathbf{c}$
  $\mathbf{r} = \mathbf{u}_i - \mathbf{U}\mathbf{c}$
  $p_i = \arg\max |\mathbf{r}|$
  $\mathbf{p} \leftarrow \{\mathbf{p}, p_i\} \in \mathbb{R}^{n \times i}$, $\mathbf{P} \leftarrow [\mathbf{P}, \mathbf{e}_{p_i}]$, $\mathbf{U} \leftarrow [\mathbf{U}, \mathbf{u}_i]$
end for

- **Note:** At each step, p_i is chosen to limit the growth of an error constant, which indicates how well the selected indices represent the full data. Algorithm 1 amounts to the index selection scheme of LU with partial pivoting applied to $\mathbf{U}_k$.
- **Variants:** Q-DEIM [4] chooses indices $\mathbf{p}$ via a pivoted QR factorization of $\mathbf{U}_k^T$.
- **Fast implementation:** DEIM can be applied quickly in an iterative fashion to process real-time event data.

Data-driven online event location via DEIM: Once an event is detected from the online data, measurements $\mathbf{Y}$ taken one second after the event are collected and approximated by the optimal rank$-k$ approximation $\mathbf{Y}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{V}_k^T$.

DEIM or Q-DEIM is applied to $\mathbf{U}_k$ in order to identify a representative set of buses for the event that can be used to locate its source.

Choosing k**:** Typically, one chooses k to capture a certain amount of variance of the data. Given some predetermined threshold $0 \ll \tau < 1$, choose the smallest integer k so that $\sum_{i=1}^k \sigma_i / \sum_{i=1}^n \sigma_i \geq \tau$.

Event Simulation Analysis

Set-up: Event simulation data of the IEEE-68 bus test system was obtained from [5] and generated using MATLAB's Power Systems Toolbox (PST).

Event simulation data for 67 three-phase (TP) short-circuit events is used for analysis. 1 second of voltage magnitudes are recorded after the event occurs and normalized against the pre-event behavior of the system.

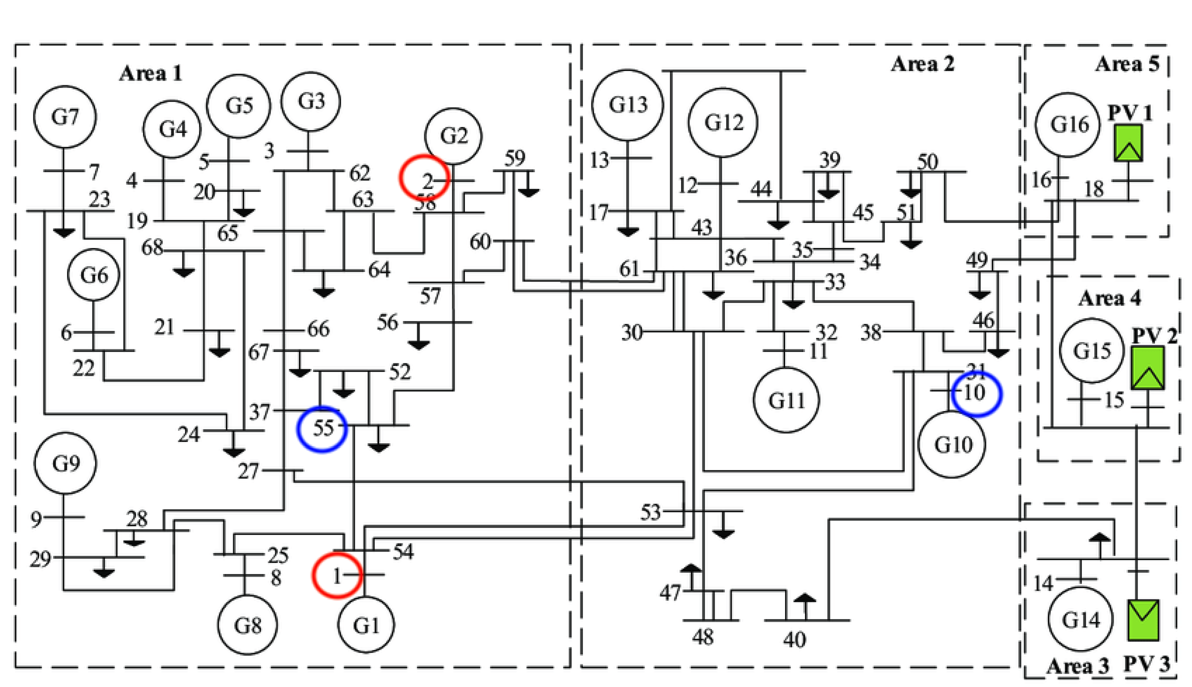

Figure 1: Model of the IEEE-68 bus system with locations of specific events reported in Figure 2 highlighted.

Event Simulation Examples

Example 1: Three phase-fault of the line connecting bus 1 and bus 2.

Example 2: Line-to-line fault connecting bus 10 and bus 55.

TP (buses 1 and 2)	1	2	3
ℓ_i^k	1	47	48
DEIM	1	2	62
Q-DEIM	1	2	62

LTL (buses 10 and 55)	1	2	3
ℓ_i^k	10	12	13
DEIM	10	55	29
Q-DEIM	10	55	29

Figure 2: The $k = 3$ buses chosen by leverage scores, DEIM, and Q-DEIM. In both scenarios DEIM and Q-DEIM correctly identify the two buses associated with the faulted line.

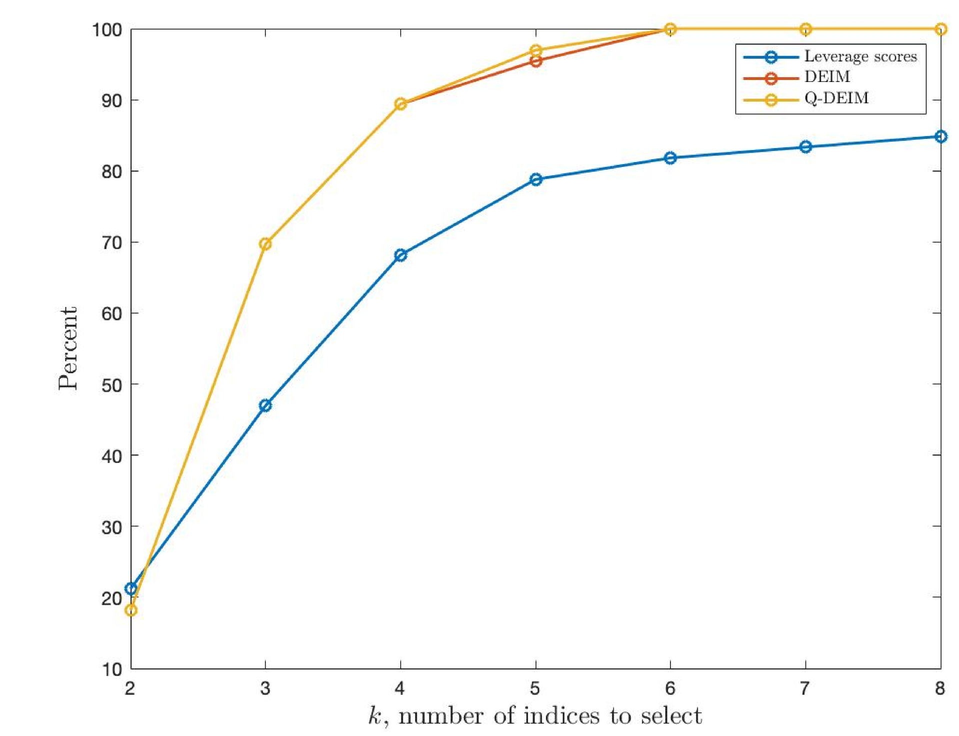

Figure 3: Percentage of event simulations in which the indicated method correctly identified both buses associated with the fault in the first k indices for $k = 2, \ldots, 8$.

Future Work

DEIM for online event detection: We plan to apply DEIM to the columns of $\mathbf{V}_k$ to find representative "timestamps", and detect when the event occurred.

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