Change in Oil Price Dynamics

Xin Jin*

Department of Economics, Southern Methodist University (SMU)

Abstract This paper studies the behavior of crude oil spot and futures prices. Oil prices, particularly spot and short-term futures prices, appear to have switched from I(0) to I(1) in early 2000s. To better understand this apparent change in persistence, a factor model of oil prices is proposed, where the prices are decomposed into long-term and short-term components. The change in the persistence behavior can be explained by changes in the relative volatility of the underlying components. Fitting the model to weekly data on WTI prices, the volatility of the persistent shocks increased substantially relative to other shocks. In addition, the risk premiums in futures prices have changed their signs and become more volatile. The estimated net marginal convenience yield using the model also shows changes in its behavior. These observations suggest that a dramatic fundamental change occurred in the period from 2002 to 2004 in the dynamics of the crude oil market.

Keywords: Crude Oil Spot Price; Crude Oil Futures Price; Futures Curve; Structural Change; Oil Price Shocks.

JEL Classification Numbers: Q4, G13, C38.

*Address: 301 Umphrey Lee, 3300 Dyer Street, TX 75275, e-mail: xjin@smu.edu. The author is grateful to Nathan Balke, Tom Fomby and Mine Yücel for advices and suggestions. All errors and mistakes are the author’s.
1 Introduction

Crude oil price has been highly volatile during the past four decades. However, oil price dynamics seem to be characterized by different features during different periods. Figure 1 displays U.S. monthly average price of imported crude oil from 1974 to 2011. From the mid-1970s to mid-1980s, large sudden spikes in the price of oil were reversed with small decreases over the following few years. Later, large upswings or downswings in oil price were then followed by a reversal to some average level within months. In late 2000s, oil price started to experience up- and down-swings of larger scale which were no longer reversed completely. The figure suggests that until early 2000s, oil price appears to be stationary with some mean level. This feature becomes less evident in the 2000s.

Crude oil futures prices also suggest a change in the dynamics of oil prices. Figure 2 plots the term structure of crude oil futures prices, or the futures curves, on different dates along with spot price at weekly frequency from 1987 to 2011. In early part of the sample, the futures curves show strong “convergence” to some constant level. Not only is the spot price mean-reverting, but it is also expected to be so by the market. Later on, the futures curves are much flatter, and appear to shift with spot price.

There is other evidence suggesting a change in oil price dynamics. Askari and Krichene (2008) find the oil price dynamics during 2002 to 2006 is characterized by more frequent and intense price jumps, causing oil market not to settle around a mean. Parsons (2009) documents a change in crude oil price term structures accompanied with a change in investment strategy in financial industry. Chevillon and Riffart (2009) argue that part of the recent changes in oil prices cannot be explained by their price model of market fundamentals. Kaufmann (2011) applies the price model of market fundamentals in Kaufmann et al. (2008) to oil prices in recent years and finds similar decrease in the explanatory ability of the model as Chevillon and Riffart (2009).

Oil prices are constantly affected by different shocks in the market. The short-term shocks usually do not affect the long-term expectation of oil price, as the short-term shocks would
dissipate over time, and only have temporary effects. Ideally, we would like to study the price dynamics in structural terms and relate the price fluctuations to movements in supply and demand. However, since equilibrium prices already reflect all the information about movements in market fundamentals, being able to identify the long-term and short-term driving forces of prices also shed light on the understanding of oil price fluctuations. Indeed, theoretically persistent and temporary shocks could generate different price fluctuations, as shown by Dvir and Rogoff (2010).

Examining oil futures prices provide additional sources for uncovering the long-term and short-term components in prices. Competitive storage theory and no-arbitrage in futures market imply difference between crude oil spot price and expected spot price should just cover the interest cost and storage cost, while the difference between crude oil futures price and expected spot price should reflect a risk premium. These relations among different prices of crude oil suggest an unobserved components model for the spot price, where it is decomposed into long-term and short-term components. This model is in the spirit of Schwartz and Smith (2000) two factor model of commodity prices. However, this model allows for time-varying risk premiums.

In this paper, I document the observed change in the crude oil price dynamics in terms of a factor model of the prices. First, I formally test for the change in the price dynamics. The change can be described as a change in persistence. A series of tests developed by Harvey et al. (2006) support a switch from I(0) to I(1) in the spot and shorter-maturity futures prices. The change in persistence is then studied within an unobserved components model of oil prices following the no-arbitrage principle. Fitting data to this model, I find that the persistence change observed in crude oil prices could be explained by an increase in the relative volatility of the long-term shocks starting from early 2000s. The risk premiums in futures prices have become more volatile too. This finding is consistent with the change in the dynamics of the term structure of oil prices documented in Parsons (2009), and is in line with the significant changes in risk premium documented in Hamilton and Wu (2011). The
net marginal convenience yield estimated using the model have become lower on average. This complements Kaufmann (2011) where the author argues for an increase in the inventory level since 2003.

The plan of the paper is as follows. Section 2 provides an overview of the data, describes the statistical tests adopted and presents the results. Section 3 develops the factor model of crude oil prices and discusses the estimation results. Section 4 presents the ability of the model to forecast crude oil prices. Section 5 concludes.

2 Crude Oil Prices Data Overview

The price data are WTI spot and futures prices as they are all quoted on the same crude oil commodity, and presumably would be affected by the same market disturbances. The maturities of the futures contracts are of 6 months, 12 months and 18 months to cover short-, mid- and long-term futures prices. The longest maturity is 18-months because most of the short-run fluctuations in prices dissipate within this horizon (Herce et al. (2006)), thus it may be far enough to provide information of the long-term component. Another consideration is the availability of futures price data as the trading of longer-term futures contracts were rare on NYMEX during 1980’s. Overall 18-month futures price would be a balance between an enough term-length for uncovering long-term expectation of oil prices and a meaningful length of time series data. I work with the daily closing prices at the end of each week, starting from the week ending on March 31, 1989 to the week ending on July 22, 2011.

2.1 Test for Change in Oil Price Dynamics

A preliminary check of the mean-reversion in oil price levels using ADF test shows that the null hypothesis of a unit root cannot be rejected for any of the price series (Table 1). For first difference of prices, the null is rejected at 1/% significant level.

---

1For example, weekly observations of 12-month futures price started in March 1989. 18-month futures contracts first started trading in September 1989, but were not traded weekly until 1995.
This results on overall persistence behavior of crude oil prices could still be consistent with the observation of earlier periods of mean-reversion in prices, as data series with a change from I(0) to I(1) contains a unit root. A series of tests developed by Harvey et al. (2006) (HLT henceforth) fit the testing purpose here well. Modified from the break point tests proposed by Kim (2000) and Busetti and Taylor (2004), HTL is a series of ratio-based tests built on the unit root testing methodology of Kwiatkowski et al. (1992). Compared to their predecessors, the HLT test statistics maintain “the same critical values regardless of whether the process is I(0) or I(1) throughout” (Harvey et al. (2006)). Thus, instead to requiring an a-priori stand regarding the null hypothesis\(^2\), HLT has a null hypothesis of constant persistence (either I(0) or I(1)) against an alternative of a change in persistence. More specifically, it tests the null against I(1) to I(0), I(0) to I(1), and the case where the direction of the change under the alternative is unspecified. Furthermore, the time of change point can be estimated as part of the testing procedure. More details of this methodology are provided in Appendix A.

### 2.2 Test Results

Harvey et al. (2006) propose two ways of modification on Kim (2000) and Busetti and Taylor (2004), which they name as “m-modified” and “m min-modified”. Results from both are presented in Table 2. In either modification, three sets of test statistics are reported in order to handle different directions of the change: the first set $MS$, $ME$ and $MX$ are to test the null hypothesis of constant persistence against the alternative of changing from I(0) to I(1) ($H_{01}$ henceforth), the second set $MS^R$, $ME^R$ and $MX^R$ against a change in the opposite direction (I(1) to I(0), $H_{10}$ henceforth), and the last set $MS^M$, $ME^M$ and $MX^M$ against a change of unknown direction. Within each set, $MS$ is a maximum-Chow-type test, $ME$ a mean score test, and $MX$ a mean-exponential test.

\(^2\)For example, Kim (2000) tests for a change from I(0) to I(1) against the null hypothesis of I(0). However, the test tends to confuse the case of a change in persistence (I(0) to I(1)) with the case of no change in persistence but under the wrong a priori null hypothesis of persistence (true process is actually I(1)). As a result, it suffers from the tendency to over-reject the null.
Like other unit root tests, HLT requires assumption about the trend behavior of the tested data. Here the prices are assumed to have no linear trend.  

Another issue is that these tests cannot deal with missing observations in data. As a result, 18-month futures price is truncated starting from the week ending in August 11, 1995, as complete weekly data is only available from that point onward. HLT searches over a potential range for possible change points in persistence. The test is set to use 20% - 80% of the sample period for all prices.

As Table 2 shows, for WTI spot price, HLT “m-modified” and “m min-modified” tests both strongly support the existence of a persistence change from I(0) to I(1): in both modifications, the constant persistence null is rejected in favor of the alternative $H_{01}$ at 1% significance level (Table 2a), and cannot be rejected in favor of alternative $H_{10}$ at any significance level (Table 2b). The last set of three statistics for testing for a change regardless of direction are also significant at 1% level (Table 2c), consistent with the conclusion from the first two sets of test statistics. The results for the 6-month and 12-month futures prices are quite similar.

The results for the 18-month futures price are mixed. “m-modified” test statistics continue to support a change from I(0) to I(1) in 18-month futures price like other prices, but “m min-modified” test statistics cannot reject the null of constant persistence.

Overall, crude oil spot price and futures prices with shorter maturity terms then should be considered as having switched from low persistence to high persistence during the sample period.

### 2.3 Break Point Estimation

The break point in each price series is estimated following the method suggested in Kim (2000) and adopted by later authors. The ratio Kim (2000) suggest is calculated over the 

---

3 The prices are also tested assuming linear trend existence. The different assumption doesn’t change the conclusion of the persistence tests.

4 The test results and estimated break points are pretty robust to the potential range setting. A range of 10% - 90% setting generates the same results.
potential range for the break, and the maximum ratio indicates the most possible break point if the null is rejected in favor of $H_{01}$. Figure 3 plots the ratio for each price series over the potential range for the break. And the plotting for spot, 6-month and 12-month futures prices actually have multiple local maximums, and two of them are very close in value. The most potential break points are in July 1998 and November 1999. Spot price has a third local maximum in around August 2002 that is also close to its global one. This could be caused by limited ability of the methodology to handle changes in intercepts in the data generating process. The estimated break points according to the global maximum of the ratios are reported in Table 2d.

3 Unobserved Components Model of Crude Oil Prices

The break point tests confirm statistically a change in the persistence behavior of crude oil prices, and provide potential dates for such a change. However, since each price series is treated individually, valuable information from the relationship among different prices to help understand the underlying price generating process is unused.

In this part, an unobserved components model is constructed following the no-arbitrage principle. The model is estimated by maximum likelihood, and the unobserved components are estimated using the Kalman filter. In addition to its ability to uncover the unobserved factors affecting crude oil prices, this model and its estimation methodology allows for missing observations in the data. Unobserved components models could take full advantage of the information in the futures prices with longer maturity terms, even when the data on those data on those futures contracts are incomplete in the early part of the sample periods.

\footnote{For 18-month futures price, the null cannot be rejected. However the ratios can still be calculated. Not suprisingly, the “break point” is estimated to be the beginning date of the potential range for the break.}
3.1 A Model of Spot Price and Futures Prices

Economic theory provides guidance for understanding the dynamics of crude oil prices. The competitive storage theory considers the arbitrage between the spot oil trading and future spot oil trading. In equilibrium, the difference between the spot price and the expected future spot price should be exactly equal to the storage cost and benefit, as well as interest cost over time. The theory for futures contracts considers the arbitrage between the crude oil futures market and the spot market, and states that the futures prices should be based on the expected future spot price and the market perception of the risk premium associated with the contracts.

The no-arbitrage of the competitive storage holding requires the following in equilibrium:

\[ p_t = E_t(p_{t+1}) - C_t^* \]  

where \( p_t \) is the spot price at time \( t \), and \( C_t^* \) is the net cost of carry incorporating all the interest cost, storage cost, and benefit from holding inventory\(^6\). \(-C_t^*\) is also referred to as “net convenience yield”. While not observable, its level may be closely related to the inventory level theoretically. When inventory is low, there’s a high possibility of stock-out, and the benefit from holding inventory would be high, resulting in lower \( C_t^* \). Short-term fluctuations that change inventory would ultimately be reflected in \( C_t^* \). \( E_t(S_{t+1}) \) on the other hand would reflect the market perceived expectation of oil prices. This is likely to reflect the long-term component in prices.

In order to reflect the concept implied by the competitive storage theory, I assume the spot price is driven by two unobserved components. The long-term component reflects the long-run expectation of the spot price, and the short-term component reflects short-term deviations from this long-run expectation. The spot price generating process is summarized by the following equations:

\[^6\text{More detailed notation would be: } P_t = E_t(P_T)/(1 + r_t) - C_t. \text{ Rewrite } C_t^* = C_t + r_t/(1 + r_t)E_t(P_T)\]
\begin{align*}
p_t &= \tau_t + c_t + \epsilon^p_t \quad & \epsilon^p_t \sim N(0, \sigma^2_p) \\
\tau_t &= \tau_{t-1} + \epsilon^\tau_t \quad & \epsilon^\tau_t \sim N(0, \sigma^2_\tau) \\
c_t &= \rho_1 c_{t-1} + \rho_2 c_{t-2} + \epsilon^c_t \quad & \epsilon^c_t \sim N(0, \sigma^2_c)
\end{align*}

where \( \tau_t \) is the long-term component, \( c_t \) is the short-term component, and \( \epsilon^p_t \) is an idiosyncratic noise term in spot price.

Futures market also implies a no-arbitrage condition:

\[ f_{t+1}^t = E_t(p_{t+1}) + RP_t \]  

(5)

where \( f_{t+1}^t \) is the price at time \( t \) for a contract maturing at \( t + 1 \), and \( RP_t \) is a term incorporating all the risk premiums associated with the futures contracts. Again, neither \( E_t(S_T) \) or \( RP_t \) is observable. An equation for futures price with maturity terms \( T \) follows through naturally:

\begin{align*}
f_T^t &= E_t(p_T) + \mu_T^{rp} + \beta_T r_p + \epsilon^{f_T}_t \\
r_p &= \rho_{rp} r_{p,t-1} + \epsilon^{r_p}_t \quad & \epsilon^{f_T}_t \sim N(0, \sigma^2_{f_T}) \\
&\quad & \epsilon^{r_p}_t \sim N(0, \sigma^2_{r_p})
\end{align*}

(6)

(7)

Here the risk premium term \( RP_t \) is modeled as \( \mu_T^{rp} + \beta_T r_p \): \( \mu_T^{rp} \) is a constant measuring the term-dependent average risk premium level; \( r_p \) is an AR(1) term measuring the time-varying part in the risk premium, and is loaded into the futures prices with term-dependent loading factor \( \beta_T \). This allows us to handle futures prices with different maturity terms, and also to allow for a flexible time-varying risk premiums. \( \epsilon^{f_T}_t \) is an idiosyncratic noise term in the futures price.

The two sets of equations from the arbitrage conditions hold simultaneously in market
equilibrium. Equations (2), (3), (4), (6) and (7) should be estimated as a system with observed $p_t$ and $f_t^T$. This model is in the spirit of the factor models like Schwartz and Smith (2000) and differs from it in the way the risk premium is modeled. In the following section, the cases of a constant risk premium and time-varying risk premium are both explored.

The proposed unobserved components model of crude oil prices can be rewritten into a state-space form. The long-term component $\tau_t$, short-term component $c_t$ described by equation (3) and (4), and the white noise in risk premium $r_{p_t}$ serve as the unobserved states underlying the price series. The state equation can be rewritten as the following in matrix form:

$$
\begin{bmatrix}
\tau_t \\
c_t \\
c_{t-1} \\
r_{p_t}
\end{bmatrix}
= F
\begin{bmatrix}
\tau_{t-1} \\
c_{t-1} \\
c_{t-2} \\
r_{p_{t-1}}
\end{bmatrix}
+ G \ast 1 + Z \nu_t
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \rho_1 & \rho_2 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & \rho_{rp}
\end{bmatrix}
\begin{bmatrix}
\tau_{t-1} \\
c_{t-1} \\
c_{t-2} \\
r_{p_{t-1}}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
\epsilon^\tau_t \\
\epsilon^c_t \\
\epsilon^{rp}_t
\end{bmatrix} \quad (8)
$$

Spot price equation and futures price (with maturity term $T$) equation can be rewritten as, respectively:

$$
p_t = \tau_t + c_t + \epsilon^p_t = \begin{bmatrix}
1 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\tau_t \\
c_t \\
c_{t-1} \\
r_{p_t}
\end{bmatrix}
+ \epsilon^p_t \quad (9)
$$
\[
\begin{align*}
  f_t^T &= E_t(P_T) + \mu_{rp}^T + \beta_T r_p t + \epsilon_t^{f^T} \\
  &= \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} (F)^T \begin{bmatrix}
  \tau_t \\
  c_t \\
  c_{t-1} \\
  r_p t
\end{bmatrix} + \beta_T r_p t + \mu_{rp}^T + \epsilon_t^{f^T} \\
  &= (\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} (F)^T + \begin{bmatrix} 0 & 0 & 0 & \beta_T \end{bmatrix}) \begin{bmatrix}
  \tau_t \\
  c_t \\
  c_{t-1} \\
  r_p t
\end{bmatrix} + \mu_{rp}^T + \epsilon_t^{f^T}
\end{align*}
\]

Equation (9) and (10) can be fit into the observation equation as follows:

\[
\begin{bmatrix}
  f_t^T \\
  p_t
\end{bmatrix} = H \begin{bmatrix}
  \tau_t \\
  c_t \\
  c_{t-1} \\
  r_p t
\end{bmatrix} + A \ast 1 + w_t
\]

\[
= \begin{bmatrix}
  1 & 1 & 0 & 0 \\
  1 & 1 & 0 & 0 \\
  1 & 1 & 0 & 0
\end{bmatrix} (F)^T + \begin{bmatrix} 0 & 0 & 0 & \beta_T \end{bmatrix} \begin{bmatrix}
  \tau_t \\
  c_t \\
  c_{t-1} \\
  r_p t
\end{bmatrix} + \begin{bmatrix}
  \mu_{rp}^T \\
  0
\end{bmatrix} + \begin{bmatrix}
  \epsilon_t^{f^T} \\
  \epsilon_p^T
\end{bmatrix}
\]

One interesting observation from earlier persistence test is that, the null of consistent persistence in 18-month futures price cannot be rejected using "m min-modified" tests, but can be rejected using "m-modified" tests, while such null for spot price and shorter-term futures prices is rejected in favor of a change from I(0) to I(1). The proposed unobserved
components model offers an explanation. Futures prices are less affected by the short-term shocks, like temporary supply disruptions. The longer the maturity term, the more the short-term shocks would dissipate, leaving the futures prices to be more like a random walk. In other words, the reason for the test results could be that 18-month is more of a random walk through out the sample period than spot or shorter-maturity futures prices. In addition, the longer the maturity term, the more the futures price would be of a random walk like the long-term component, as the effects of the short-term shocks dissipate anyway for the longer-maturity futures prices.

With this factor model of crude oil prices (8) and (11), each price has a non-stationary component as well as a stationary component underlying. When \( \sigma^2 \), the variance of the innovation to the non-stationary long-term component, is relatively large, the prices could be “dominated” by the long-term component \( \tau_t \), and appear to be non-stationary; when \( \sigma^2 \) is small, the prices could be dominated by the short-term component \( c_t \), and appear to be stationary. Thus, depending on the type of prevailing shocks in the market and their sizes relative to others, the prices may show different persistence properties during different periods of time, as we observe in the data.

In terms of the relationship among different prices, this model implies that the difference between the spot and futures prices mostly comes from the short-term component and the risk premium. When \( \sigma^2 \) is relatively large, the difference tends to be large, resulting in the “converging” pattern of the futures curves in Figure 2. When the size of \( \sigma^2 \) decreases relatively, all prices would appear to be similar, and the futures curves become flatter.

Therefore the model provides possible explanations for most features of the observed crude oil price dynamics. We conjecture that there might be an increase in the relative size of the variance of shocks to the long-term component, \( \sigma^2 \) over time, leading to the change in persistence observed in the data.
3.2 Estimation Results

This system is estimated with the same data series tested for persistence change, except in full length. Here it is possible to use 18-month futures price data with missing observations because in the state space model, the missing data problem can be easily dealt with: when there is missing data at time $t$, the updating matrix in Kalman filter to calculate the likelihood is set to be zero, meaning that there is no information from the data to help updating the states at $t$.

3.2.1 Constant versus Time-varying Risk Premiums

The model is estimated first with constant risk premiums. The maturity-term-dependent loading factor $\beta$ is set to zero for all three futures prices. We also estimate the model with time-varying risk premiums. Here we allow correlation between risk premiums and long-term and short-term components, respectively. The reason is, risk premiums reflect the market perceived risk associated with the futures contracts, and such perception could very possibly be affected by the long-term and short-term components. As Table 3 shows, the time-varying risk premiums improves the estimated likelihood of the model significantly with a log-likelihood ratio of 2315.41.

3.2.2 Break Point versus No Break Point

The crude oil prices show apparent change in persistence. Statistical tests also confirm the change and suggest potential candidates for the break point. We also search for a possible break point with the unobserved components model. We model the break as a one-time change of all the parameters at some point. The maximum likelihood of the model with a break at some point can be estimated over a potential range, and the break point with which the estimated maximum likelihood is the highest would be the most likely time for the change. Note that with this unobserved components model, the estimation of two sets of parameters prior to and after the break is not equivalent to splitting the data into two and
estimating the model using two subsamples separately like in the structural break literature. The estimation of an unobserved components model also involves filtering the unobserved states from the available observed data. When the model allowing for a break is estimated using the complete data sample, the initial state for the model after the break is filtered using the complete history of the observed data prior to the break. When the data is split and the model estimated separately, filtering the initial state for after the break doesn’t take the earlier data into consideration. Different initial states affect the likelihood function, and it would be more “natural” to use the whole sample to estimate the model allowing for a break.

In the search for the most likely break point, the model is estimated with time-varying risk premiums that are correlated to the long-term and short-term components. Figure 4 plots the estimated maximum likelihoods over the potential range for the change. The search using the unobserved component also suggests multiple candidate dates, with a highest estimated maximum likelihood of 1471.25 during the week ending on August 13, 2004. Another local maximums correspond to 21-Mar-2003. These suggested dates are not exactly the same as those from Kim (2000), but already quite close. Kim (2000) methodology suggests two close possible dates from 1998 to 2000, and the unobserved components model suggests two close possible dates from 2003 to 2004 instead. In the following estimation and comparison of the model with a break and one without a break, the change is set to take place during the week ending on August 13, 2004 following the suggestions from the unobserved components model.

Estimates from both cases are reported in Table 4. The estimated log likelihood, AIC and BIC all suggest that the model with a break-point is an over-all better fit of the data: the model allowing for a break-point has significantly higher log likelihood, and lower AIC and BIC statistics. The point estimates of most parameters in both cases are significant the 99% significance level. In the model allowing for a break point, only the second coefficient $\rho_2$ in the AR(2) process for the short-term component, the mean risk premium level for
6-month futures contracts after the break and the covariances between the time-varying risk premium and the long-term and short-term components are insignificant. The covariances between the risk premium and other two components are significant at 99% level, but very close to zero nonetheless. In both cases, the measurement errors in the observation equations for futures prices are very close to zero.

To reconcile with the persistence change in prices, we conjecture earlier that the variances of the long-term component shock would increase relatively to the short-term component. In the model allowing for a break, the point estimate of $\sigma^2_\tau$, the variance of the long-term component shocks, is 0.1738 before the break, and 8.3621 after the break. $\sigma^2_c$, the variance of the short-term component shocks, increases from 1.0009 to 8.3197. $\sigma^2_{rp}$. Both components become much more volatile after the break, but the volatility of $\sigma^2_\tau$ increases by nearly a factor of 50. The random-walk long-term component becomes “larger” in later part of the sample, contributing to the change from I(0) to I(1) in data.

The changes in autoregressive coefficients for the short-term component could also be responsible for its relatively decreased importance in affecting prices. We check the roots for the AR(2) process prior to and after the break point. In the model with a break, the eigenvalues for the AR(2) coefficient matrix are [0.9791; 0.0008] prior to the break, [0.9917; −0.0168]. When loaded into the prices, the loading factor of short-term components $[c_t; c_{t-1}]$ prior to the break are [1; 0] for spot price, [0.6023; −0.0005] for 6-month futures price, [0.3331; −0.0003] for 12-month futures price, and [0.2005; −0.0002] for 18-month futures price; the loading factor after the break are [1; 0] for spot price, [0.8048; 0.0135] for 6-month futures price, [0.6371; 0.0107] for 12-month futures price, and [0.5214; 0.0088] for 18-month futures price. The changes in the factor loading also seem to indicate more persistent behavior over time.

The filtered long-term and short-term components using the model with a break are shown in Figure 5. The trend component is plotted against the spot price in gray. It is easy to see that the variances of both long-term and short-term components increase substantially. During the earlier half of the sample period, there’s clearly a level around which the spot
price fluctuates. This level reflects the market consensus of the “normal” price. After the break however, spot price appears to “drift” with no constant level they tend to return to.

### 3.2.3 Implied Risk Premiums Using the Model Allowing for a Break

Figure 5 shows the third filtered state $r_{P_t}$ and recovered risk premiums in the futures prices using the model. The increase in the volatility of $r_{P_t}$, which is the time-varying component in the risk premiums (Eqs. (6) and (7)), is also striking. An interesting observation is the change in the recovered risk premiums prior to and after the break. The risk premiums are systematically negative prior to the break. The risk premiums in 12-month and 18-month futures prices are never positive. Also the risk premiums are more negative with maturity terms for 6-month and 12-month futures, though the risk premium in 18-month futures get less negative than those with shorter maturities occasionally. Overall, the time path of 6- and 12-month futures risk premiums are more similar, with very stable 18-month futures risk premiums.

**Keynes** (1930) theory of risk premium proposed that if hedgers in the futures market are net short (i.e. producers of the physical commodity) seeking price protection, then in order to entice speculator into taking the offsetting long positions, risk premium would be negative as a reward. And the longer the maturity term, naturally the higher the risk associated with the futures contract, thus the reward for bearing the risk would tend be larger in size. When the hedgers are net long (i.e. buyers), risk premium would be positive. It would follow from the theory that the sign and magnitude of risk premium would logically be related to the distribution of hedgers. **Alquist and Gervais** (2011) estimate the correlation between the changes in net positions with changes in oil prices and find a positive correlation for non-commercial firms (speculators) and a negative correlation for commercial firms (hedgers). In other words, when oil price increases, long positions of speculators increase on the net, while short positions of hedgers increase on the net. The negative risk premiums prior to the break indicate speculators on average would long crude oil futures contracts. **Parsons** (2009)
documents the strategy of a long position in short-maturity futures contracts as a common practice in the financial industry.

After the break, risk premiums become much more volatile. One additional key change is that the risk premiums are no longer negative all the time. The average of 6-month futures risk premium after the break is slightly positive, and over half of the times, the premium is positive. 12-month and 18-month futures prices also turn positive for a considerable number of times. It would follow from the theory that, the increase in the volatility of long-term component is also accompanied by dramatic changes in the futures market. The longer-term futures risk premiums are still negative on average, but the 18-month futures risk premium is much more volatile than before. Rather, all risk premiums become more similar. Parsons (2009) observes a striking change in the financial investment strategy from being long in a short-maturity contract to being long in a long-maturity contract. The estimated risk premiums using the unobserved components model fit well with the observation.

### 3.2.4 Contributions of Different Components to Oil Prices

Figure 7 and 8 plot the contributions of long-term, short-term and risk premium components to each price series against the oil price. Judged by their relative scales, though all three components become more volatile after the break, the long-term component contribute the most to the recent upswings in all prices. A comparison of the 2007 to 2008 oil price spike to the one in 1990 show striking differences. During the first Gulf War in 1990, the peak of spot price was accounted for by short-term disruptions that doesn’t last long. To the contrary, the increase in spot price during 2007 to 2008 was mostly driven by long-term changes of market dynamics that affect all oil prices to similar extent. Though risk premiums also indicate changes in futures market, their contribution to prices in absolute terms are limited compared to the other two components.
3.2.5 Implied Convenience Yield Using the Model Allowing for a Break

It’s possible to recover the net marginal convenience yield using the unobserved components model. Recall equation (1) from the competitive storage theory. The difference $E_t(P_{t+1}) - P_t$ should cover the interest cost and marginal storage cost and benefit incurred holding inventory. In the literature, the negative of this difference is also referred to as “net convenience yield”. The idea is that though there’s interest cost and storage cost, holding inventory may lower the possibility of stock-out, or insure the continuous operation of the production facility, and this benefit can be greater than the cost. As a result, even when the spot price is high, firms still keep inventory. Figure ?? shows estimated net convenience yield at 1-week, 1-month and 6-month horizons using the model allowing for a break.

Similar to the net convenience yield for heating oil estimated in Pindyck (1994), the plotting during the earlier half of the sample period in Figure ?? fluctuates around zero with occasional spikes. After the break, the estimated marginal convenience yield using the unobserved components model tend to be more negative, especially at 6-month horizon. This indicates high level of inventory since intuitively marginal convenience yield is decreasing in inventory level. Kaufmann (2011) documents a significant increase in private US crude oil inventories since 2004. Figure ?? plots the net marginal convenience yield against U.S. commercial inventory during the sample period.

4 Forecasting Performance

To assess the ability of the unobserved components models to forecast crude oil prices, we generate one-month and 6-month ahead out-of-sample forecasts of the spot price staring from Jan 1, 2010\textsuperscript{7}. We compare the forecasts to those implied by futures prices, a random walk in Figure 11 and 12. Table 5 presents evaluation of the forecasting using $MSE$, $MAE$ and $ME$. For both forecasting horizons, the unobserved components models outperform

\textsuperscript{7}When forecasting using the model with a break, the break point is re-estimated using only the in-sample data.
futures prices and a random walk. Apparently, the information from multiple futures prices improves the forecasting performance. Interestingly the model without a break does a little better than the one with a break, perhaps because the post-break sample for estimate is a little shorter due to the need to preserve data for the out-of-sample forecast.

5 Conclusion

Crude oil prices could follow different dynamics during different periods. Specifically there are periods of low persistence and apparent mean-reversion and periods of high persistence with little mean-reversion. We test for the observed change in persistence in crude oil prices, and confirm that there is a change from I(0) to I(1) in spot price and shorter-term futures prices. Longer term futures prices tend to be of higher persistence with lower volatility. We fit an unobserved components model with time-varying risk premium for the crude oil prices to weekly data over past 22 years. The model incorporates information from both spot and futures prices to help understand crude oil price dynamics and suggests a likely break point in Aug 2004. When a break point is allowed, the estimates of the model show that the variance of the shocks to the long-term component increases substantially after the break. The relative volatility also increases compared to short-term component. The observed change in persistence can be related to the volatilities of underlying shocks. The volatility of risk premiums also increases, indicating changes in the futures market, but the contribution to oil futures prices are limited compared to other two components. Estimated net marginal convenience yield also suggests a change in market fundamentals during the same period. In terms of forecasting, the model outperforms futures prices alone and a random walk in our forecasting horizons.

These results strongly suggest changes in the crude oil market that are able to generate long-term effects on prices as the driving force of recent oil price increases. This suggestion points to rigid world oil supply, highly price inelastic oil demand driven by strong world
economy growth, possible inventory-holding capacity and others as the candidate factors to help understand the price dynamics and recent price fluctuations. The next step would be to study the mechanism under which the market fundamentals are able to generate more volatile long-term effects on prices, which would explain the increase in long-term volatility we document in this paper.
A Test Methodology for Change in Persistence

Like Harvey et al. (2006), the following model underlying the test statistics is considered:

\[ y_t = x_t' \beta + v_t \]  

(12)

\[ v_t = \rho v_{t-1} + \epsilon_t, \quad t = 1, 2, ..., T \]  

(13)

with \( v_0 = 0 \). \( y_t \) is the crude oil price. The vector \( x_t \) is assumed to satisfy the mild regularity conditions of Phillips and Xiao (1998), and one example is the \( k \)th order polynomial trend, \( x_t = (1, t, ..., t^k)' \). Since the data seems to exhibit no apparent trend, \( x_t' = 1 \). \( \epsilon_t \) is a mean zero process satisfying the familiar \( a \)-mixing conditions of Phillips and Perron (1988), with strictly positive and bounded long-run variance.

Harvey et al. (2006) considers four hypotheses. The first is that \( y_t \) is \( I(1) \) throughout the sample period, denoted \( H_1 \). Both unit root and local to unit root behavior are covered, and \( \rho_t \) is set such that \( \rho_t = 1 - \alpha / T, \alpha > 0, t = 1, ..., T \). The second hypothesis, denoted \( H_{01} \), corresponds to a change from \( I(0) \) to \( I(1) \) at time \( [\tau_* T] \), i.e., \( \rho_t = \rho, |\rho| < 1, \) for \( t \leq [\tau_* T] \) and \( \rho_t = 1 - \alpha / T, \alpha \geq 0, \) for \( t > [\tau_* T] \). Here \( \tau_* \) is the unknown change-point proportion in \( \Lambda = [\tau_l, \tau_u] \), an interval in \( (0,1) \) which is symmetric around 0.5, with \( \tau_l \) and \( \tau_u \) represent the lower and upper bound for \( \tau_* \). The third hypothesis, denoted \( H_{10} \), corresponds to a change from \( I(1) \) to \( I(0) \) at time \( [\tau_* T] \), i.e., \( \rho_t = 1 - \alpha / T, \alpha \geq 0, \) for \( t \leq [\tau_* T] \) and \( \rho_t = \rho, |\rho| < 1, \) for \( t > [\tau_* T] \). The last hypothesis, denoted \( H_0 \), is that \( y_t \) is \( I(0) \) throughout, i.e., \( \rho_t = \rho, |\rho| < 1, \) \( t = 1, ..., T \) in the model.

The tests in the next section are based on the following ratio developed by Kim (2000) and a technique originally devised by Vogelsang (1998). The ratio \( K_{[\tau T]} \) proposed by Kim (2000) is:
\[ K_{[\tau T]} = \frac{(T - [\tau T])^{-2} \sum_{t=[\tau T]+1}^{T} (\sum_{i=[\tau T]+1}^{t} \hat{\nu}_{i,\tau})^2}{[\tau T]^{-2} \sum_{i=1}^{[\tau T]} (\sum_{t=1}^{i} \hat{\nu}_{i,\tau})^2} \quad (14) \]

where \( \hat{\nu}_{i,\tau} \) in the denominator is the residual from the OLS regression of \( y_t \) on \( x_t \), for \( t = 1, \ldots, \tau T \). In this paper, since \( x_t = 1 \), i.e., \( \hat{\nu}_{i,\tau} = y_t - \bar{y}_t(\tau) \) with \( \bar{y}_t(\tau) \) being the average of \( y \) up to \( \tau T \). Similarly, \( \hat{\nu}_{i,\tau} \) in the numerator is the OLS residual from the OLS regression for \( t = \tau + 1, \ldots, T \).

To test a change in persistence (\( H_0 \) against \( H_{01} \)), Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004) consider the following three statistics, based the sequence of the ratio \( \{K_{\tau T}, \tau \in \Lambda\} \) defined above:

\[ MS = T_*^{-1} \sum_{t=[\tau T]}^{[\tau_0 T]} K_t, \quad (15) \]

\[ ME = ln\{T_*^{-1} \sum_{t=[\tau T]}^{[\tau_0 T]} exp(0.5K_t)\}, \quad (16) \]

\[ MX = \max_{t \in [\tau T], \ldots, [\tau_0 T]} K_t, \quad (17) \]

where \( T_* \equiv [\tau T] - [\tau_0 T] + 1 \).

To test \( H_0 \) against \( H_{10} \), Busetti and Taylor (2004) propose further tests based on the sequence of reciprocals of \( K_t \). They define \( MS^R, ME^R, MX^R \) as the analogues of \( MS, ME, MX \) with \( K_t \) replaced by \( K_t^{-1} \) throughout. In order to test against an unknown direction of change, they propose \( MS^M = \max[MS, MS^R], ME^M = \max[ME, ME^R], MX^M = \max[MX, MX^R] \).

Harvey et al. (2006) adopt a technique by Vogelsang (1998) to modify the above nine test statistics. The approach is largely the same for all the tests, and so here only the
modification of MS is presented in particular detail. Namely, they define

\[ MS_m = \exp(-bJ_{1,T})MS, \]  

(18)

where \( b \) is a finite constant and \( J_{1,T} \) is \( T^{-1} \) times the Wald statistic for testing the joint hypothesis \( \gamma_{k+1} = \ldots = \gamma_9 = 0 \) in the regression

\[ y_t = x_t'\beta + \sum_{i=k+1}^{9} \gamma_i t^i + \text{error}, \quad t = 1, \ldots, T. \]

where \( k \) is the highest order of the polynomial trend in \( x_t \). In this paper, \( x_t = 1 \) and thus \( k = 0 \). Similarly \( MS^R_m = \exp(-bJ_{1,T})MS^R \), and \( MS^M_m = \exp(-bJ_{1,T})MS^M \). The corresponding mean-exponential (\( ME, ME^R, ME^M \)) and maximum statistics (\( MX, MX^R, MX^M \)) can be modified in exactly the same fashion. Collectively, the tests are referred to as "m-modified".

A variant of the above modification procedure, which, according to Harvey et al. (2006), is "more natural to consider when testing against \( H_{01} \)", is obtained by replacing \( J_{1,T} \) by \( J_{\text{min}} = \min_{r \in A} J_{1,[rT]} \), where \( J_{1,[rT]} \) is the inverse of the effective sample size \( \hat{T}^{-1} = (\tau T)^{-1} \) times the Wald statistics for testing the joint hypothesis \( \gamma_{k+1} = \ldots = \gamma_9 = 0 \) in the regression

\[ y_t = x_t'\beta + \sum_{i=k+1}^{9} \gamma_i t^i + \text{error}, \quad t = 1, \ldots, \lfloor \tau T \rfloor. \]

That is, the newly modified statistic is defined

\[ MS_{m \text{ min}} = \exp(-bJ_{\text{min}})MS. \]

However, to test against \( H_{10} \), \( MS^R_{m \text{ min}} \) is now defined

\[ MS^R_{m \text{ min}} = \exp(-bJ^R_{\text{min}})MS^R \]

where \( J^R_{\text{min}} = \min_{r \in A} J_{[rT],T} \) and \( J_{[rT],T} \) is the inverse of the effective sample size \( \hat{T}^{-1} = \)
\((T - \tau T)^{-1}\) times the Wald statistics for testing the joint hypothesis \(\gamma_{k+1} = \ldots = \gamma_9 = 0\) in the regression

\[ y_t = x_t' \beta + \sum_{i=k+1}^{9} \gamma_i t^i + \text{error}; \quad t = [\tau T] + 1, \ldots, T. \]

As regards to the test against an unknown direction of change, \(MS^M\) is defined

\[ MS^M_{m \min} = \exp(-b \min [J_{\min}, J^R_{\min}]) MS^M. \]

This set of modified test statistics are referred to as "m min-modified".

For all test statistics from two versions of the modifications, critical values (both finite sample and asymptotic) and values for \(b\) are provided in Table 1 and Table 2 in Harvey et al. (2006), respectively.

To estimate the break point, the ratio Kim (2000) suggests is calculated over the potential range \([\tau l T, \tau u T]\) for the break:

\[ \Lambda_{[\tau T]} = \frac{(T - [\tau T])^{-2} \sum_{t=[\tau T]+1}^{T} \hat{e}_t)^2}{[\tau T]^{-2} \sum_{t=1}^{[\tau T]} \hat{e}_t^2} \]  

When the null is rejected in favor of \(H_{01}\), the estimated break point is:

\[ \tau_{01} = \arg \max_{t \in [\tau l T], \ldots, [\tau u T]} \Lambda_{[\tau T]} \]  

When the null is rejected in favor of \(H_{10}\), the estimated break point is:

\[ \tau_{10} = \arg \min_{t \in [\tau l T], \ldots, [\tau u T]} \Lambda_{[\tau T]} \]
### Table 1: Overview of Crude Oil Prices Persistence

**(a) level**

<table>
<thead>
<tr>
<th>Series</th>
<th>ADF t-statistic</th>
<th># of lags</th>
<th>ADF t-statistic</th>
<th># of lags</th>
<th>ADF t-statistic</th>
<th># of lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI spot</td>
<td>−0.703</td>
<td>1</td>
<td>−0.705</td>
<td>2</td>
<td>−0.799</td>
<td>3</td>
</tr>
<tr>
<td>6-month</td>
<td>−0.339</td>
<td>1</td>
<td>−0.393</td>
<td>2</td>
<td>−0.550</td>
<td>3</td>
</tr>
<tr>
<td>12-month</td>
<td>−0.120</td>
<td>1</td>
<td>−0.186</td>
<td>2</td>
<td>−0.329</td>
<td>3</td>
</tr>
<tr>
<td>18-month</td>
<td>−0.314</td>
<td>1</td>
<td>−0.362</td>
<td>2</td>
<td>−0.441</td>
<td>3</td>
</tr>
</tbody>
</table>

**(b) first difference**

<table>
<thead>
<tr>
<th>Series</th>
<th>ADF t-statistic</th>
<th># of lags</th>
<th>ADF t-statistic</th>
<th># of lags</th>
<th>ADF t-statistic</th>
<th># of lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI spot</td>
<td>−24.20***</td>
<td>1</td>
<td>−19.07***</td>
<td>2</td>
<td>−15.34***</td>
<td>3</td>
</tr>
<tr>
<td>6-month</td>
<td>−23.26***</td>
<td>1</td>
<td>−17.83***</td>
<td>2</td>
<td>−14.99***</td>
<td>3</td>
</tr>
<tr>
<td>12-month</td>
<td>−22.98***</td>
<td>1</td>
<td>−17.80***</td>
<td>2</td>
<td>−15.14***</td>
<td>3</td>
</tr>
<tr>
<td>18-month</td>
<td>−19.28***</td>
<td>1</td>
<td>−15.26***</td>
<td>2</td>
<td>−13.02***</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 2: Test for Change in Persistence in Crude Oil Prices

(a) Test: constant persistence against H_{01}

<table>
<thead>
<tr>
<th>Series</th>
<th>T</th>
<th>MS_m</th>
<th>ME_m</th>
<th>MX_m</th>
<th>MS_{min}</th>
<th>ME_{min}</th>
<th>MX_{min}</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI spot</td>
<td>1165</td>
<td>56.67***</td>
<td>52.32***</td>
<td>192.94***</td>
<td>117.60***</td>
<td>206.26***</td>
<td>538.47***</td>
</tr>
<tr>
<td>6-month</td>
<td>1165</td>
<td>57.40***</td>
<td>46.13***</td>
<td>220.40***</td>
<td>194.0***</td>
<td>424.6***</td>
<td>1187.2***</td>
</tr>
<tr>
<td>12-month</td>
<td>1165</td>
<td>54.03***</td>
<td>23.86***</td>
<td>140.22***</td>
<td>163.95***</td>
<td>207.11***</td>
<td>688.67***</td>
</tr>
<tr>
<td>18-month</td>
<td>833</td>
<td>41.96***</td>
<td>22.18***</td>
<td>130.88***</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

(b) Test: constant persistence against H_{10}

<table>
<thead>
<tr>
<th>Series</th>
<th>T</th>
<th>MS^R_m</th>
<th>ME^R_m</th>
<th>MX^R_m</th>
<th>MS^R_{min}</th>
<th>ME^R_{min}</th>
<th>MX^R_{min}</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI spot</td>
<td>1165</td>
<td>0.13</td>
<td>0.12</td>
<td>1.20</td>
<td>0.06</td>
<td>0.05</td>
<td>0.51</td>
</tr>
<tr>
<td>6-month</td>
<td>1165</td>
<td>0.06</td>
<td>0.04</td>
<td>0.52</td>
<td>0.1</td>
<td>0.1</td>
<td>0.6</td>
</tr>
<tr>
<td>12-month</td>
<td>1165</td>
<td>0.03</td>
<td>0.01</td>
<td>0.23</td>
<td>0.05</td>
<td>0.05</td>
<td>0.56</td>
</tr>
<tr>
<td>18-month</td>
<td>833</td>
<td>0.19</td>
<td>0.06</td>
<td>0.47</td>
<td>0.25</td>
<td>0.16</td>
<td>0.85</td>
</tr>
</tbody>
</table>

(c) Test: constant persistence against change of either direction

<table>
<thead>
<tr>
<th>Series</th>
<th>T</th>
<th>MS^M_m</th>
<th>ME^M_m</th>
<th>MX^M_m</th>
<th>MS^M_{min}</th>
<th>ME^M_{min}</th>
<th>MX^M_{min}</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI spot</td>
<td>1165</td>
<td>43.33***</td>
<td>36.43***</td>
<td>130.20***</td>
<td>72.96***</td>
<td>112.77**</td>
<td>303.94**</td>
</tr>
<tr>
<td>6-month</td>
<td>1165</td>
<td>39.05***</td>
<td>27.43***</td>
<td>125.34***</td>
<td>105.3***</td>
<td>196.0***</td>
<td>570.8***</td>
</tr>
<tr>
<td>12-month</td>
<td>1165</td>
<td>33.52***</td>
<td>12.53***</td>
<td>69.68***</td>
<td>64.67***</td>
<td>63.86***</td>
<td>225.94***</td>
</tr>
<tr>
<td>18-month</td>
<td>833</td>
<td>25.98***</td>
<td>11.62***</td>
<td>64.87***</td>
<td>17.29***</td>
<td>13.19***</td>
<td>56.98***</td>
</tr>
</tbody>
</table>

(d) Estimated Break Point

<table>
<thead>
<tr>
<th>Series</th>
<th>T</th>
<th>break-point</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI spot</td>
<td>1165</td>
<td>1999/11/12w</td>
</tr>
<tr>
<td>6-month futures</td>
<td>1165</td>
<td>1998/07/03w</td>
</tr>
<tr>
<td>12-month futures</td>
<td>1165</td>
<td>1998/07/31w</td>
</tr>
<tr>
<td>18-month futures</td>
<td>833</td>
<td>1998/10/16w</td>
</tr>
</tbody>
</table>

*Note: (i) *, ** and *** denote rejection at the 10%, 5% and 1% levels, respectively; (ii) \( \tau_l \) is set to be 20%, \( \tau_u \) 80%; (iii) The notation \( MS_{m} \) indicates that the modified test \( MS_{m\min} \) was run at the 5% level;
Table 3: Estimated Likelihood of Different Model Setting

<table>
<thead>
<tr>
<th></th>
<th>no break</th>
<th>break</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant risk premium</td>
<td>-2580.02</td>
<td>-1558.04</td>
</tr>
<tr>
<td>time-varying risk premium</td>
<td>-264.61</td>
<td>1470.91</td>
</tr>
</tbody>
</table>

Note: the break point for the “break” case is set at Aug 13, 2004.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior to Break Point</th>
<th>Post Break Point</th>
<th>No Break Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break-point</td>
<td>2004/08/13w</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>1470.9</td>
<td>-264.6</td>
<td></td>
</tr>
<tr>
<td>Akaike (AIC) criterion</td>
<td>-2873.9</td>
<td>597.2</td>
<td></td>
</tr>
<tr>
<td>Bayesian (BIC) criterion</td>
<td>-2701.8</td>
<td>649.2</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.98 (0.018)**</td>
<td>0.97 (0.018)**</td>
<td>0.98 (0.037)**</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>-7.78e-4 (0.017)</td>
<td>0.017 (0.018)</td>
<td>-0.02 (0.036)</td>
</tr>
<tr>
<td>$\sigma^2_r$</td>
<td>0.17 (0.010)**</td>
<td>8.36 (0.653)**</td>
<td>3.68 (0.159)**</td>
</tr>
<tr>
<td>$\sigma^2_c$</td>
<td>1.00 (0.064)**</td>
<td>8.32 (0.924)**</td>
<td>0.97 (0.091)**</td>
</tr>
<tr>
<td>$\sigma^2_{rp}$</td>
<td>0.01 (0.010)**</td>
<td>0.63 (0.090)**</td>
<td>1.17e-4 (1.034e-5)**</td>
</tr>
<tr>
<td>$\sigma^2_p$</td>
<td>0.88 (0.044)**</td>
<td>0.49 (0.095)**</td>
<td>1.33 (0.067)**</td>
</tr>
<tr>
<td>$\sigma^2_{f6}$</td>
<td>4.23e-7 (1.31e-8)**</td>
<td>2.32e-8 (7.304e-10)***</td>
<td>4.86e-7 (1.506e-8)***</td>
</tr>
<tr>
<td>$\sigma^2_{f12}$</td>
<td>7.75e-9 (2.428e-9)**</td>
<td>1.85e-7 (5.835e-9)**</td>
<td>9.03e-8 (2.858e-9)**</td>
</tr>
<tr>
<td>$\sigma^2_{f18}$</td>
<td>5.81e-6 (8.694e-7)**</td>
<td>0.24 (0.019)**</td>
<td>0.03 (0.002)**</td>
</tr>
<tr>
<td>$\mu_{rp,6}$</td>
<td>-0.91 (0.083)**</td>
<td>-0.22 (0.437)</td>
<td>0.21 (0.456)</td>
</tr>
<tr>
<td>$\mu_{rp,12}$</td>
<td>-1.60 (0.134)**</td>
<td>-1.62 (0.607)**</td>
<td>0.10 (0.707)</td>
</tr>
<tr>
<td>$\mu_{rp,18}$</td>
<td>-1.98 (0.181)**</td>
<td>-2.92 (0.742)**</td>
<td>0.03 (0.845)</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>1.06 (0.213)**</td>
<td>1.07 (0.051)**</td>
<td>-12.10 (0.153)**</td>
</tr>
<tr>
<td>$\beta_{18}$</td>
<td>0.34 (0.561)**</td>
<td>1.01 (0.087)**</td>
<td>-28.36 (0.071)**</td>
</tr>
<tr>
<td>$cov_{r,fp}$</td>
<td>0.00 (0.009)</td>
<td>-0.09 (0.049)*</td>
<td>0.01 (0.001)**</td>
</tr>
<tr>
<td>$cov_{c,fp}$</td>
<td>-0.00 (0.009)</td>
<td>-0.09 (0.049)*</td>
<td>0.00 (4.819e-4)**</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>0.95 (0.003)**</td>
<td>0.91 (0.025)**</td>
<td>0.97 (0.005)**</td>
</tr>
</tbody>
</table>

Note: (i) Standard errors are in parentheses; (ii) *, **, and *** denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

*the loading factor for 6-month futures prices is normalized to be 1.
Table 5: **Out-of-sample Forecast Performance**

(a) One-month ahead Out-of-sample Forecast

<table>
<thead>
<tr>
<th>forecaster</th>
<th>MSE</th>
<th>MAE</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>model with break</td>
<td>48.95</td>
<td>5.53</td>
<td>0.45</td>
</tr>
<tr>
<td>model without break</td>
<td>46.20</td>
<td>5.39</td>
<td>0.18</td>
</tr>
<tr>
<td>1-month futures price</td>
<td>49.45</td>
<td>5.57</td>
<td>0.69</td>
</tr>
<tr>
<td>random walk</td>
<td>50.94</td>
<td>5.66</td>
<td>0.94</td>
</tr>
</tbody>
</table>

(b) 6-month ahead Out-of-sample Forecast

<table>
<thead>
<tr>
<th>forecaster</th>
<th>MSE</th>
<th>MAE</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>model with break</td>
<td>143.23</td>
<td>9.68</td>
<td>5.57</td>
</tr>
<tr>
<td>model without break</td>
<td>139.79</td>
<td>9.47</td>
<td>5.48</td>
</tr>
<tr>
<td>6-month futures price</td>
<td>140.39</td>
<td>9.49</td>
<td>5.53</td>
</tr>
<tr>
<td>random walk</td>
<td>193.63</td>
<td>11.47</td>
<td>9.09</td>
</tr>
</tbody>
</table>
Figure 1: Crude oil spot price

Source: EIA.
Figure 2: WTI spot price with futures curves

Figure 3: Kim (2000) ratios for estimating the break point
Figure 4: Estimated maximum likelihoods with different break points
Figure 5: Filtered long-term and short-term components with 90% CI

Note: The grey curve in Figure (a) is WTI spot price. The vertical line represents the time of break-point, 20040813w.
(a) time-varying component in risk premium: $r_{pt}$

(b) risk premiums

Figure 6: Risk premiums in different futures prices

Note: The vertical line represents the time of break-point, 20040813w.
Figure 7: Contributions of different components to oil prices

Note: The vertical line represents the time of break-point, 20040813w.
Figure 8: Contributions of different components to oil prices

Note: The vertical line represents the time of break-point, 20040813w.
Figure 9: Net marginal convenience yields at different horizons

Note: The vertical line represents the time of break-point, 20040813w.
Figure 10: Net marginal convenience yield (6-month) and end-of-week U.S. commercial inventory

Note: The vertical line represents the time of break-point, 20040813w.
Figure 11: One-month ahead out-of-sample forecasts
Figure 12: 6-month ahead out-of-sample forecasts
References


