CREDIBLE SIGNALS OF THE RELEASE OF NEW VERSIONS

Abstract

Prices can credibly signal whether a durable-goods monopolist will offer an improved good in future periods. When the future release of a new version is private information, a monopoly seller will reveal a new version of a good with a high price, and failure to develop and market a new version with a lower price than in full information. A firm would be willing to pay more to innovate when consumers are uncertain than if they are informed ex ante, because a failure to innovate is punished by a low equilibrium price. Consumers’ uncertainty about innovation intensifies an unsuccessful innovator’s Coasian problem and increases consumer welfare. (JEL D82, L12, L15)
I. Introduction

Many products are marketed some time after they have been developed. For instance, a new smartphone may be developed and tested well before it is available for purchase. A new car model might be prototyped and driven months before it comes to market. Indeed, a firm will often have several generations concurrently in a development “pipeline;” consumers may know a product is in that pipeline without knowing its progress towards the market. Because the outcome of such development and testing is not visible to outsiders, the firm knows whether it can market a product before consumers do. In the interim consumers are uninformed when, and whether, the new product will be marketed to them.

Economists have long recognized that present demand for durable goods depends on future prospects, such as the future price or the introduction of an improved version. The existence of such an improved version is often the seller’s private information. If consumers anticipate a new version, some will defer purchases until the new version is available, and the seller will face low present demand. Disclosing a new version is not in a monopoly seller’s interest. A seller without a new version would also like to communicate that fact to consumers.

While a firm with a successful product innovation can credibly disclose the new version with relative ease, a firm that has failed to develop a new product could have great difficulty convincing consumers that it has no new version. In such situations, it is important to examine whether prices can inform consumers whether a new version will be marketed in the near future.

In a recent paper, Choi et al. (2010) develop a model with a risky product development process, in which reputation effects allow cheap-talk announcements to signal successful product development to consumers. An announcement induces consumers to delay or divert purchases to the announcer’s coming product, which increases the

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1 For instance, Bulow writes, “rational consumers recognize that the monopolist will not consider their interests [in maintaining a high resale price] in the second period and will adjust the price they are willing to pay for first-period purchases accordingly,” (Bulow, 1986, 733).
announcer’s profit and can be welfare-improving.\footnote{This is in contrast to the antitrust concerns which birthed the “vaporware” literature; see for instance Bayus et al. (2001).} In their model the announcer and her non-innovative rivals supply the original good competitively. Because of this an announcement does not reduce profit on the current version. The present paper will focus on markets where the seller has substantial market power before innovating, so announcing a successful product development has a cost in terms of current revenue. Further, because we attend to prices as signals, this model may also apply to markets where a long-run relationship between consumers and firms may not be sustainable.

Another objective of this paper is to examine the welfare effects of mandatory disclosure. A government policy mandating disclosure of any new version is an alternative means of informing consumers. As long as firms are not willing to lie to the government, such a policy would credibly communicate whether a new version of a durable is coming. Mandatory disclosure would bring about full information, since consumers are informed about any improved future version before observing the price of the current version.

In this paper, we explore a two-period model of a durable-good monopoly in which consumers are initially uninformed whether there will be a new version of the current good. The monopoly seller is privately informed whether he will be able to market an improved version in the second period. In the first period, consumers must decide whether to purchase the good now, or wait for an upgraded version. They may be able to infer from the price of the current version whether the seller will bring a new version to market. It is important to see, first, whether prices can serve as signals, and then how welfare compares between any signaling outcome and a full-information benchmark corresponding to mandatory disclosure. We will show that there is a unique equilibrium supported by ‘intuitive’ beliefs; in this equilibrium the seller’s private information about an improved version is fully revealed to consumers. We will also show that prices are lower, and total surplus is higher, under incomplete information than if consumers were initially informed whether there is a new version. In other words, a policy mandating
disclosure of improved versions would reduce welfare in the product market.

It is also important to compare the effects of these information structures on the seller’s incentive to invest in innovation in the first place. In this model a failure to bring a new product to market is punished with lower profit under incomplete than full information, so a firm has a stronger incentive to succeed in bringing a good to market than when consumers are informed \textit{ex ante} about a new version.

This model abstracts from retailing or distribution and inventory effects. The point in question is not whether consumers will infer from a sale of inventories that the firm is releasing a new version, but whether consumers can infer from the regular price of the current version that there will or will not be an improved version at some future date.

Prior works in durable-goods monopoly describe the problem of a firm competing with its own output over time. Though Coase’s conjecture\textsuperscript{3} \textit{per se} is not robust, this “Coasian problem” remains important to models of durable-goods monopoly and oligopoly.\textsuperscript{4} Improving a good and recapturing high-willingness-to-pay consumers is one way firms ameliorate this problem.\textsuperscript{5} In the literature, a new version is sometimes\textsuperscript{6} offered to past customers at a different price than first-time buyers; we will concentrate on the case of anonymous consumers who can trade the good in a competitive secondary market. In contrast to the present paper, most prior research assumes that the existence of a new version is common knowledge.

Waldman (1996) examines a different time-consistency problem faced by a durable-goods monopolist. Consumers’ first-period demand depends on the value of holding the original version in the second period, and thus on the degree of an improvement in the good. So, the marginal benefit from investing in research and development (R&D)

\textsuperscript{3}See Coase (1972).

\textsuperscript{4}For instance, see Bulow (1986), Bagnoli et al. (1989) or Deneckere and Liang (2008); Waldman (2003) surveys.

\textsuperscript{5}Leasing is another such, which we will consider below in section IV. Selling prevails in many durable goods markets, perhaps because a renter faces moral hazard in her use of the rented good. As we are interested in the effects of private information about future versions, we focus on the case of a monopolist who sells his output.

\textsuperscript{6}See Fudenberg and Tirole (1998) on when this is in the seller’s interest, and Fishman and Rob (2000) on its implications for R&D investment.
is different before and after first-period sales. Also in a full-information setting, Nahm (2004) examines the same problem when past customers may be identified. This paper abstracts from time inconsistency in the seller’s willingness to pursue research and development, to focus on the effect of private information on product development.

There is a large literature in which price signals current product quality.\(^7\) This paper explores a different problem than this quality-signaling literature, since in this paper the incomplete information is about an improved future good; the present quality is known. The seller uses current prices to signal whether there will be an improved version in the future. Though in the quality-signaling literature \textit{ex ante} profit may be decreasing in the probability of possessing a high-quality good, the reverse is true here: the seller is willing to pay to have some probability of marketing a new version. Also in contrast, a seller with an improved version has an incentive to imitate the first-period price of a seller without one.

In the next section I introduce the model and present two full-information cases as benchmarks. Section three contains the main results, first for separating equilibria, for pooling equilibria, and then the implications for welfare and the firm’s incentive to innovate. I then examine the leasing outcome in section four. The fifth section discusses these results. All proofs are in the appendix.

\section{The model}

In this section I present a two-period model of durable-goods monopoly where consumers face uncertainty about whether the monopoly seller will bring a new good to market. For the present, we will not model the product development process, but take its success or failure as given.\(^8\) I assume the good is perfectly durable over the two

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\(^7\)See for instance Bagwell and Riordan (1991). In a recent addition, Banerjee and Soberman (2013) develop a two-period model of a durable-good monopoly where current and future quality are the seller’s private information. In that model the first-period price signals the quality of the current \textit{and} future goods. In their signaling outcome, the incentive to invest in quality improvement is no different than in full information. In addition to the differences from the quality-signaling literature noted here, Banerjee and Soberman restrict parameters so that the incentives to mimic found in this paper cannot emerge.

\(^8\)We will address the seller’s investment problem in subsection III.C.
periods of the model, and the ‘size’ of the potential innovation is common knowledge.9

A. Primitives

There is a mass of consumers normalized to one who purchase either one or zero units of the good. Their marginal valuation $\theta$ of quality is distributed uniformly on the unit interval. A consumer with valuation $\theta$ receives utility $\theta s$ from holding a good of quality $s$ for one period, and consumer surplus from purchasing the same at a price $p$ is $\theta s - p$. The quality of the existing good is normalized to one. That of the new good, if introduced, is commonly known to be $v$ greater than one.10 If a consumer holds a good of the new quality, no utility is gained by also holding an unimproved version of the good.11 In the second period, consumers can buy or (if holding one) sell a unit of the good on a competitive used good market. Consumers and the seller have a common discount factor $\delta$ between zero and one. The monopolist cannot commit in the first period to a second-period price.12 The seller produces either quality of the good at zero unit cost.

Call the type of the seller with an improved version of the good $S$ and the type whose efforts to market a new version have failed $F$. In the first period, nature chooses the type of the seller from a distribution that assigns probability $\alpha$ to type $S$ and probability $1 - \alpha$ to type $F$. This distribution is commonly known. The seller observes his type and sets a price for the existing good. Consumers form beliefs given this price whether the monopoly seller is of type $S$ and choose to purchase or not to purchase at this price. In the second period, the seller’s type is publicly revealed. If the seller is

9Though this may not be reasonable for experience or credence goods, some goods have easily known and comparable dimensions of quality, e.g., the fuel efficiency of an automobile, or the capacity of a data storage device.

10Firms can and do introduce lower-quality versions of a good in order to price discriminate (see Deneckere and McAfee (1996)). We restrict our attention to the case of an improved good, so that the only price discrimination possible is intertemporal.

11This is reasonable for many consumer durables, such as large home appliances.

12Relaxing this assumption eliminates the failed type’s Coasian problem but does not remove the successful type’s incentive to imitate the type without a new version. If it not also possible for the seller without a new version to commit not to sell a new version of the good, we will be back in a case of incomplete information, with signaling substantially similar to that below.
of type $F$, i.e., the seller has no improved version of the good, he sets a price for the unimproved good, and consumers choose to purchase or not. If the seller is of type $S$, he sets a price for the improved good. Consumers who did not buy in the first period choose whether to buy the new version, or buy the old version on the used-good market, or make no purchase. Consumers who did buy in the first period choose whether to retain the old good, or sell their old unit on the used market and buy the new.

B. The full-information benchmark

We solve the model by backward induction in the special cases where the firm is publicly known to be of type $F$ and type $S$ at the beginning of period one. In addition to benchmarks for the case of incomplete information, these outcomes may be interesting in their own right. If there were a regulation requiring disclosure of a new version, a firm of type $S$ would disclose the new version, and a firm of type $F$ would not, and consumers would learn the type of the firm before observing any price. Then one of the full-information outcomes below would follow, depending on the realized type of the firm. Below I compare consumer and producer welfare under this regime to that when prices inform consumers of the seller’s type.

Take the case where in the first period, the firm is publicly known to be of type $F$. Let $\theta^F_1$ be the type of the consumer just indifferent to buying in the first period, and $\theta^F_2$ that of the indifferent buyer in the second. All consumers of a higher type will strictly prefer to buy, so first-period demand is $1 - \theta^F_1$. Second-period demand will be the residual demand from period one, given by $\theta^F_1 - \theta^F_2$. Since we chose the indifferent consumer, this second-period buyer will satisfy $\theta^F_2 = p^F_2$ where $p^F_2$ is the seller’s second-period price. Consumers expect that the seller will set a price to maximize second-

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13 It might seem that the monopolist should sell the old good in the second period, though each unit sold dilutes the profit from the improved good. Fudenberg and Tirole (1998) show that, when the cost difference between the original and improved good is small enough, selling the old good in the second period is not optimal for a monopolist with an innovation. Old models of a good are often “killed off” in actual markets, as with new cars or personal computers. Any fixed costs associated with marketing the old concurrently with the new can also be avoided this way.

14 While such a mandate might seem extreme, truth in advertising rules may already prevent a firm with an innovation from affirmatively advertising that it does not possess one.
period profit\(^{15}\) given first-period sales. That is, \(p^*_2 = \arg \max_p p(\theta^*_F - p) = \theta^*_F / 2\). Then the first-period indifferent consumer will satisfy \(\theta^*_F = p^*_F / (1 + \delta / 2)\) and the monopolist will accordingly set a first-period price to maximize profit

\[
\Pi^F = p^*_F (1 - \theta^*_F) + \delta (\theta^*_F / 2)^2.
\]

This full-information first-period price is \(p^*_F = (2 + \delta)^2 / 2(4 + \delta)\). The ‘failed’ monopolist will have full-information profit \(\Pi^F = (2 + \delta)^2 / 4(4 + \delta)\).

\[\text{Figure 1: Resale Price Determination}\]

In the case where the monopolist is publicly known in the first period to have succeeded, consumers know that in the second period, some or all of the old goods will be resold in a competitive secondary market. Let the first-period indifferent buyer have marginal value for quality \(\theta^*_S\) and the indifferent buyer of the new good have \(\theta^*_S\). The indifferent second-period buyer will have the same utility from selling the old version\(^{16}\) at the used-good price \(p_u\) and buying the new, or holding the old, that

\(^{15}\) There can be no secondary market here, since first-period buyers prefer holding the good to selling it to consumers with, by hypothesis, a lower willingness to pay.

\(^{16}\) Alternatively, a consumer who did not purchase the good in the first period is indifferent between buying in the new and used goods markets; this second derivation yields an identical second-period demand, though in equilibrium the producer chooses prices so that the first formulation is realized.
is, $\theta_S^2 v - p_2 + p_u = \theta_S^2$, so $\theta_S^2 = (p_2 - p_u)/(v - 1)$. Every consumer who buys in both periods will also sell the used good.\footnote{For ease of exposition, I show a secondary market with no transaction costs or any degradation of the good from one period of use. For a small enough friction (less than $\theta_u$ given above), the sales of new and used goods will be decreased, but the results below will be qualitatively preserved. A large enough friction would make trade in the used good unattractive to any used-good buyers, and so reduce first-period buyers’ willingness to pay for the new good. The firm could then adopt a trade-in policy and discriminate between new purchasers and re-purchasers of the good, as with Fudenberg and Tirole (1998) semi-anonymous consumers.} For every second-period price of the new good $p_S^2$, we can find the used-good price which clears the used-good market. This will be given by $p_u = \theta_S^1 + \theta_S^2 - 1$; see Figure 1 and note that $\theta_u = p_u$, since the marginal used-good buyer is indifferent to buying or not. Then, $\theta_S^2$, the marginal valuation for quality of the indifferent buyer of the improved good, will satisfy $\theta_S^2 = (p_S^2 - 1 + \theta_S^1)/v$. The innovator $S$ will then choose the best second-period price, $p_S^2 = \arg \max_p (p/v) ((v - 1) - p + \theta_S^1) = ((v - 1) + \theta_S^1)/2$. Then the first-period indifferent buyer satisfies $\theta_S^1 = (p_S^1 + \delta(v - 1)/2v) / (1 + \delta/2 + \delta(v - 1)/2v)$ and the monopolist will set a first-period price to maximize profit, as given in equation 2.

$$\Pi^S = p_S^1(1 - \theta_S^1) + (\delta/4v)(\theta_S^1 + v - 1)^2$$

(2)

That full-information price will be $p_S^F = ((2 + \delta)^2v + \delta(4 + 3\delta)(v - 1))/2(3\delta(v - 1) + (4 + \delta)v)$. The successful monopolist’s full-information profit will be

$$\Pi^S = v[3\delta^2(v - 1) + (4 + \delta)\delta v + 4]/4[3\delta(v - 1) + (4 + \delta)v].$$

We can then order the two types’ full-information prices and profits. Buying the old good now if there is a new one coming is also to buy an option to repurchase at a discount, while the certainty of second-period sales in the case of a failed-type firm necessitates a discount if anyone is to buy now rather than later.

**Lemma 1.** The first-period full-information prices are ordered $p_F < p_S$, and the full-information profit of type $S$ exceeds that of type $F$.

We should not understand this order of prices and profits by analogy to the quality-
signaling literature. A firm of type $F$ cannot gain from imitating a firm of type $S$; as we shall see below, the reverse is true. We assume that quality is observable, so the only uncertainty is whether a new version will be available in the second period. The prices here are for identical goods, and a higher price will preserve more demand for tomorrow, for either the same or an improved version; from $F$’s perspective, imitating the price of a seller with a new version will allocate too much demand to the second period.

III. Signaling Equilibrium Results

Now we return to incomplete information, where the seller’s type is not initially known to consumers in the first period. The successful innovator would like to imitate a failed innovator, but may be prevented from doing so by the failed type’s lowering his first-period price. There are a multiplicity of (Perfect Bayesian) equilibria, and we use the Intuitive Criterion of Cho and Kreps (1987) to focus on those supported by ‘reasonable’ out-of-equilibrium beliefs. There is a unique separating equilibrium which survives refinement. An innovative monopolist never has lower profit than in full information, while the failed type has a strictly lower profit.

An equilibrium survives refinement by the Intuitive Criterion if no type has a necessarily beneficial deviation\(^\text{18}\) from the equilibrium, given the out-of-equilibrium beliefs. For a given deviation from the equilibrium, those beliefs assign a probability of zero to any type for whom that action’s payoff is not better than the equilibrium payoff, even given the most favorable best response by the signaling audience. This refinement selects the separating equilibrium with the least cost from signaling, and rules out any pooling equilibria for models with two types.\(^\text{19}\)

In this case of incomplete information, consumers form beliefs $\mu(p)$ that the firm is of type $S$ when observing a first-period price $p$. Their responses to this price are captured

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\(^{18}\)That is, the payoff from this deviation is better than the equilibrium for every best response by the audience; even at worst, this type does better making this deviation than in equilibrium.

\(^{19}\)This additionally requires the types’ preferences satisfy a single-crossing condition. For more, see Cho and Kreps (1987) or Fudenberg and Tirole (1991), 446-460.
in the marginal value of the marginal first-period buyer. This will be intermediate between the full-information values of \( \theta_1 \), and given by \( \theta_1(p, \mu(p)) = (p + \mu \delta(v - 1)/2v)/(1 + \delta/2 + \mu \delta(v - 1)/2v) \). So, \( \theta^F_1 = \theta_1(p, 0) \) and \( \theta^S_1 = \theta_1(p, 1) \). Let \( \Pi^f(p, \mu(p)) \) be the profits of each type setting price \( p \) and with consumer beliefs \( \mu(p) \); its form will be like the full-information profits given above in equations 1 and 2, except that the marginal first-period buyer’s type will be \( \theta_1(p, \mu(p)) \).

### A. A separating equilibrium

Our first result is that there is no separating equilibrium where the full-information prices are charged. This is because the innovating type of the seller will always gain from imitating the type without a new version at the full-information price.

**Proposition 1.** There is no equilibrium in which a firm of type \( S \) charges the full-information price \( \bar{p}^S \) and a firm of type \( F \) charges \( \bar{p}^F \) in the first period.

Some consumers would wait to buy tomorrow from a monopolist who had a new version; if he conceals the innovation now by pricing as if there were not a new good, they will purchase now, and some of them will still want the new good once the deception is revealed in the second period. Imitating the failed type’s price in the first period allows the innovator to ‘double-dip’ on these consumers and enjoy the revenue of two sales. Since \( S \) can gain from imitating \( F \), a separating equilibrium will require \( F \) to distort his price away from \( F \)'s full-information price. In a separating equilibrium, consumer beliefs will assign a probability of zero that the firm is of type \( S \) at the price charged by \( F \) and one at the price charged by \( S \), so type is revealed in equilibrium.

Then in a separating equilibrium where \( F \) charges a price \( p \), it must be that \( \Pi^S(p, 0) \leq \Pi^S \), so the innovator is never better off imitating the failed type at \( p \) than setting the full-information price, and \( \Pi^F(p, 0) \geq \Pi^F(\bar{p}^S, 1) \) so that \( F \) has no incentive to imitate \( S \) in the first period. This will be true for prices outside the interval \((\tilde{p}, \bar{p}^+)\) shown in Figure 2. These prices are defined by \( \Pi^S(\tilde{p}, 0) = \Pi^S(\bar{p}^+, 0) = \Pi^S(\bar{p}^+, 1) \), so
that $S$ is just indifferent to imitating $F$ at these prices. These prices will be

$$
\frac{(2 + \delta)(2 + \delta)v + \delta(v - 1)}{2(4 + \delta)v + \delta(v - 1)} \pm \frac{(2 + \delta)v}{(4 + \delta)v + \delta(v - 1)} \sqrt{\frac{2\delta(v - 1)}{(4 + \delta)v + 3\delta(v - 1)}}.
$$

If consumers observe a price in $(\bar{p}, \bar{p}^+)$, they will believe it comes from $S$ with probability near one, while for prices outside this interval, they believe it comes from $F$ with high probability, as shown. Given these out-of-equilibrium beliefs, $S$ and $F$ could set many prices, but the Intuitive Criterion will select an equilibrium outcome where $F$ sets $\bar{p}$ and $S$ sets the full-information price $p^S$.

**Proposition 2.** In any separating equilibrium satisfying the Intuitive Criterion, $S$ will set the full-information price $p^S$ and $F$ set the minimally-distorted price $\bar{p}$.

Since in equilibrium type is revealed, $S$ is known to be of type $S$, and $p^S$ is the best price for $S$ when this is true. The failed type incurs the least cost from signaling when charging $\bar{p}$. While a lower price also signals type, his profit is increasing in price here, so $F$ would always benefit from an increase in price toward $\bar{p}$. Likewise, while it is possible to signal type with $\bar{p}^+$ or a higher price, it will take a larger, and therefore more costly, distortion to do so.
In the second period, the type of the seller is publicly revealed, and each type will set a second-period price to maximize profit given first-period sales. Because he is charging the same first-period price and sells the same amount of the old good as in full information, a seller of type $S$ will also set the same second-period price and sell the same quantity of the new good as in full information. A seller of type $S$ will get the full-information profit.

In contrast, a seller of type $F$ will sell a greater first-period quantity than in full information, since his price $\tilde{p}$ is below the full-information price $p^F$. Similarly to full information, $F$ will set the second-period price to maximize profit on the residual demand, and sell $\tilde{p}/(2 + \delta)$ in the second period of a separating equilibrium. This is less than full-information second-period sales of $p^F/(2 + \delta)$. The type of the seller without an innovation sells more over the two periods and makes strictly lower profit in this equilibrium than under full information.

The successful type charges his full-information price and gets the full-information profit. These are both increasing in the new quality $v$ since consumers’ willingness to pay for a new good in the second period increases with its quality $v$. As that quality increases, the consumer indifferent between purchasing now and later will have a higher marginal value of quality, so that $S$ will do best by charging the remaining consumers with higher valuations a high price. The failed type’s profit in this equilibrium is increasing in its price, and depends on $v$ only through $\tilde{p}$. This price is first decreasing
then increasing in the potential new quality; see Figure 3. If the innovation is minor, 
$F$ needs only a small distortion in price to signal type, because the successful type’s 
gain from imitation is small; but as $v$ grows more distortion is needed, since if deceived 
more consumers would repurchase from a successful innovator when the new quality 
is higher. As $v$ becomes large, the innovator’s profit from the new version grows and 
makes imitating a failed-type firm less enticing; the distortion required to deter an 
innovator from imitating accordingly becomes smaller. Since $F$’s profit is increasing in 
price near $\tilde{p}$, $F$’s profit in this separating equilibrium will also be first decreasing, then 
increasing in the potential new quality.

B. Pooling Equilibria

We now attend to pooling equilibria, where since both types set the same first-period 
price, consumers cannot update their beliefs and believe a seller setting this price is of 
type $S$ with the prior probability $\alpha$. As is usual for games with this number of types, 
no pooling equilibrium survives the Intuitive Criterion.

**Proposition 3.** No pooling equilibrium survives the Intuitive Criterion.

Take a pooling equilibrium at price $p$ when the prior probability the firm is of type 
$S$ is $\alpha$. We can construct the price $q(p, \alpha)$ for which, at best, when consumers believe 
the seller is of type $S$ with probability near zero, a seller of type $S$ is just indifferent 
between deviating to this price and getting the pooling equilibrium profit. Then a 
deviation to a price $p' = q - \varepsilon$ would yield a seller with a new version a lower profit 
than in equilibrium. So, if consumers see observe such a price, they will believe it 
comes from a seller of type $F$ with probability near one. Given this belief, a seller 
of type $F$ would benefit from deviating to $p'$ from $p$. Such an equilibrium fails the 
Intuitive Criterion.
C. The incentive to innovate

A firm which fails to innovate is punished with low profit in the separating equilibrium above; a seller’s expected profit is increasing in the probability of having a new version, so the seller would be willing to pay more to innovate, compared to full information.

Let us suppose a seller must make a fixed investment to have a given probability of innovating. Under full information, a seller who can invest an amount $f$ to innovate with probability $\alpha$ has expected net profit $\alpha \Pi^S + (1 - \alpha) \Pi^F - f$ if investing, and $\Pi^F$ if not. A risk-neutral seller will choose to invest if $f$ is less than or equal to $\alpha (\Pi^S - \Pi^F)$. Likewise, a seller facing consumers who are uninformed whether his investment succeeds or fails to produce a new version has an expected profit from investing of $\alpha \Pi^S + (1 - \alpha) \bar{\Pi}^F - f$. Then a seller under incomplete information will choose to invest when the fixed cost $f$ is less than $\alpha (\Pi^S - \bar{\Pi}^F)$. Since the seller of type $F$ has lower profit in the separating equilibrium than in full information, this is higher than the maximum investment in full information. So, a seller will have a stronger incentive to innovate when consumers are uncertain than when they are informed about a forthcoming version.

D. Welfare Analysis

Total surplus is higher in expectation under incomplete information than full information; a policy requiring disclosure of any new versions would be reduce welfare, compared to the separating equilibrium. When we take the cost of innovation into account, welfare is higher under incomplete than full information for high and low costs of innovation. This is also true for intermediate costs of innovation if there is a high enough probability of success.

We first examine surplus in the product market, then take into account the cost of innovation below. If the realized type of the seller is successful, the same quantity will sell at the same price in either case. We therefore compare welfare when the realized type does not have an innovation. The failed type will sell more in the first period,
and make higher total sales over the two periods, under incomplete information than full information. We can write product market welfare $W$ when the realized type is $F$ as the feasible surplus less deadweight loss in each period\(^{20}\) so $W_F = (1/2)(1 - \theta_1^2) + (\delta/2)(1 - \theta_2^2)$; see Figure 4. Since $F$ will set its price to make $\theta_2 = \theta_1/2$, this means $W_F = (1/2)(1 + \delta - \theta_1^2(1 + \delta/4))$. Let $\bar{W}_F$ be surplus under full information, and $\tilde{W}_F$ under incomplete information when the realized type is $F$. Then $\bar{W}_F - \tilde{W}_F = [(4 + \delta)/8](\bar{\theta} - \theta_1^F)(\bar{\theta} - \bar{\theta}_1^F)$. Since these marginal valuations are respectively $\bar{p}$ and $\bar{p}^F$ over $1 + \delta/2$ and $\bar{p} < \bar{p}^F$, the last term will be negative. Though a seller of type $F$ is worse off under incomplete than full information, consumers benefit from lower equilibrium prices, and surplus will be higher, as will expected surplus before the seller’s type is realized. A mandate to disclose a new version would reduce surplus in the product market.

If innovation has succeeded, we can write surplus similarly to $W_F$ above, as $(1/2)(1 - \theta_1^{S^2}) + (\delta v/2)(1 - \theta_2^{S^2})$ plus the whole surplus realized in the used-good market.\(^{21}\) Since $1 - \theta_1^S$ units of the old good are held by consumers, and they have a value of $\theta_u = \theta_1^S + \theta_2^S - 1$, revenue here is $\theta_u(1 - \theta_1^S)$. Consumer surplus on the used-good market is just $(\theta_2^S - \theta_u^S)^2/2 = (1 - \theta_1^S)^2/2$. Then, plugging in the equilibrium values of $\theta_1^S$, $\theta_2^S$ and $\bar{p}_1^S$, total welfare is

$$W_S = \frac{v(3\delta^2(4v - 3) + 4\delta(3v + 1) + 12)}{8(4(\delta + 1)v - 3\delta)}.$$ 

Let us suppose that innovation requires a fixed cost, $f(v)$, to be paid, in order for a new quality of $v$ to be realized with a probability $\alpha$. Using the upper bounds on the seller’s willingness to invest found above, we can define two critical values of $f$: let $\tilde{f}$ be the most a seller is willing to invest to innovate with probability $\alpha$ under

\(^{20}\)Since it is efficient for all consumers to have the good in both periods, deadweight loss in a period is just the forgone surplus of all consumers who do not hold a unit of the good. We will find welfare net of investment costs below.

\(^{21}\)Used-good buyers enjoy the consumer surplus and the revenue accrues to purchasers of the new good. Note that if $\theta_2$ is greater than $\theta_1$, some consumers will hold the original version for two periods with a second-period surplus of their type $\theta$; it is just as if these last buy the good at the prevailing price, and receive the proceeds of that sale.
full information, and \( \bar{f} \) be the most a seller under incomplete information is willing to invest to innovate with the same probability. If \( f(v) \) is below \( \bar{f} \), then a seller will invest \( f(v) \) under either information structure. Because the same amount is invested in either case, the only differences in welfare arise in the product market, and are exactly those studied above. If \( f(v) \) is above \( \bar{f} \), the seller will not invest under either information structure, and we compare welfare when the realized type is \( F \), as above.

If instead the fixed cost of innovation lies between these cutoffs, a seller is willing to invest under incomplete information, but not under full information. We will compare expected welfare, net of the fixed cost, under incomplete information to welfare if the realized type is \( F \) under full information. Suppose expected welfare under incomplete information net of the investment is higher than under full information with no investment; that is,

\[
\alpha W_S + (1 - \alpha) \tilde{W}_F - f > \bar{W}_F
\]

which will be satisfied for \( \alpha \) greater than or equal to \( \bar{\alpha} \) given below in equation 4.

\[
\bar{\alpha} = \frac{-\bar{W}_F + \tilde{W}_F - f}{\bar{W}_F - W_S}
\]

We can therefore state the following.

**Proposition 4.** If the fixed cost of innovation is below the maximum investment under full information \( \bar{f} \), or above the maximum investment under incomplete information \( \tilde{f} \), expected welfare net of the fixed cost will be higher under incomplete information than full information; if the cost \( f \) lies between them, expected welfare under incomplete information will be higher if the probability of innovating is at least \( \bar{\alpha} \), and lower otherwise.

Note that \( \bar{\alpha} \) will pass above unity for high enough levels of new quality. If \( \bar{\alpha} \) is one, \( \tilde{W}_F - W_S = \tilde{W}_F - \bar{W}_F - f \), or \( W_S - \bar{W}_F = f \). We can write that equation more

---

22This is a restrictive assumption; if further investment can increase the probability of success, a seller under incomplete information would invest more than one under full. But, the effect on welfare will be ambiguous: more investment detracts, and a higher probability of success could raise or lower expected welfare, depending on the size of the improvement in quality.
explicitly as
\[
\frac{3\delta(v - 1) \left( -3\delta^2 - 16\delta + 4 \left( \delta^2 + 5\delta + 4 \right) v - 12 \right)}{8(\delta + 4)(\delta + 1)v - 3\delta)} = f.
\]

Let \( \hat{v} \) be the quality which satisfies this for a given \( \delta \) and fixed cost \( f \). For intermediate fixed costs and new qualities greater than \( \hat{v} \), welfare would be higher under full information, with no investment, than under incomplete when the firm is pressed to innovate.

For high and low costs of innovation, a mandate to disclose a new version would reduce welfare net of the cost of innovation. It would also do so for intermediate costs, when the probability of success is high enough. Consumers are never better off for being informed \textit{ex ante} whether a new version is coming.

**IV. Leasing**

We now turn to the leasing outcome in this setting. In addition to a benchmark for the case of sales, our results on welfare and the lessor’s incentive to innovate may be of independent interest. Since the 1953 decision in \textit{U.S. v. United Shoe Machinery}, solely leasing output has been suspect on antitrust grounds. Several researchers caution that a prohibition on leasing may not improve welfare if the seller introduces “other types of social welfare distortions,” (Waldman, 1997, 84) to soften his Coasian problem, for instance reducing the durability of sold output.\(^{23}\) In contrast, we show that leasing provides higher welfare than selling for sufficiently large improvements to the good in the absence of any such mechanism. Leasing also gives the firm a stronger incentive to innovate than selling.\(^{24}\)

Coase suggested that leasing instead of selling would prevent the monopolist of a durable good from facing competition from a future self: when output is leased

\(^{23}\)See for instance Bulow (1986) or Malueg and Solow (1987). Differently, Malueg and Solow (1989) show that leasing may improve on selling when a seller’s first-period output choice deprives many high-willingness-to-pay consumers of the good. Waldman (1993) finds that leasing could free the firm from an incentive to introduce too many style changes, from a social planner’s perspective.

\(^{24}\)This is in contrast to Bond and Samuelson (1987), who find selling may induce too much investment in process, rather than product, innovation.
a consumer’s consumption choice in a period is independent of that choice in other periods, or of future consumption prospects, like an improvement in quality. In the setting considered in the present paper, we will see that leasing will separate consumers’ present consumption from their expectations about any forthcoming new version. The monopoly lessor will rent the highest available quality in each period, and enjoy profits like those of a monopolist of a non-durable good in each period; leasing effectively makes the good non-durable. Profit will be higher under leasing than selling, and when new quality is low, consumer and total surplus will be lower with leasing than with selling. When the quality is higher, the monopoly lessor’s gains in profit will swamp consumers’ loss of surplus, and total surplus will be higher than under selling.

Though they are uncertain about the future availability of an improved version, consumers’ rental choice does not depend on their expectations. Renting today does not alter their future willingness-to-pay, since the lessor recovers all units of the good. Let \( \theta_1^r \) be the marginal value of the marginal first-period renter when the monopolist charges \( r_1 \), so that \( 1 - \theta_1^r = 1 - r_1 \) consumers rent. Then the lessor will choose \( r_1 = 1/2 \) to maximize first-period profit \( r(1 - r) \).

Let \( \theta_2^s \) denote the marginal value for quality of the indifferent second-period renter; that is, \( \theta_2^s s = r_2 \), when the monopolist charges rental rate \( r_2 \) and \( s \) is the highest realized quality of the good, either one or \( v \).\(^{25}\) Then every consumer with a higher marginal value strictly prefers to rent, so second-period demand is \( 1 - r_2/s \). The monopoly lessor’s problem in the second period is \( \max_{r} r(1 - r/s) \), so that \( r_2 = s/2 \), where \( s \) is the highest quality available. The discounted present value of leasing profits will be \( (1 + \delta v)/4 \) if the monopolist has innovated, or \( (1 + \delta)/4 \) if he has not.

We find consumer and total surplus under leasing, and compare it to that under selling when the monopolist succeeds and fails to innovate. Then, we find the \textit{ex ante} welfare from leasing and selling. Consumer surplus when renting will be \( (1 - \theta_1^r)^2/2 + \)

\(^{25}\) It might seem that in the second period the producer could gain from offering both versions of the good and segmenting the market. In fact, renting the original version will so cannibalize demand for the new that, when offering both, the optimal second-period price for the original version will ensure no units of the old version are rented; then the rate and quantity of the new version are exactly as presented here. See Lemma 10 in the appendix.
δs(1 − θr2)^2/2; compare to Figure 4. When we plug in the equilibrium values, this is
(1 + δs)/8, and therefore total welfare will be (3/8)(1 + δs), when the highest available
quality is $s$. Let $W_{rS}$ be this welfare when innovation succeeds and $s = v$, and $W_{rF}$
when innovation fails and $s$ is one. Then we establish two preliminary results.

**Lemma 2.** If innovation has failed, welfare is higher under selling than leasing.

If innovation has failed, consumer surplus is higher under selling than leasing. First-
period buyers enjoy the good for two periods, and the seller sells at a low price to
consumers who would never have rented. These gains are large enough to offset the
seller’s low profit.

**Lemma 3.** If innovation has succeeded, welfare is higher under selling than leasing
when $v < 9/8$, and lower otherwise.

If innovation succeeds, welfare may be higher or lower under leasing than selling,
depending on the new quality $v$. A lessor is always better off than a seller. For low
levels of the new quality, consumers are better off buying than renting, since more
consumers buy some version of the good than would rent. The lessor’s profit grows
faster than a seller’s as $v$ increases, and the consumers who do rent gain from the
high quality; for high enough $v$, these gains more than balance the losses in consumer
surplus because of the lessor’s high price and restricted output, compared to a seller.

We can now compare \textit{ex ante} expected welfare from selling and leasing. Expected
product-market welfare when the monopolist sells is $\alpha W_S + (1 - \alpha)\tilde{W}_F$, and when he
leases, $\alpha W_{rS} + (1 - \alpha)W_{rF}$, so the difference in expected welfare is given below.

\[ \alpha [W_S - W_{rS}] + (1 - \alpha)(\tilde{W}_F - W_{rF}) \] (5)

We have just shown that the last term is always positive, and that the term in square
brackets is positive when $v$ is less than 9/8. If new quality is below this level, expected
welfare is higher under selling than leasing. If the new quality is higher, we must
account for the probability with which an improved version is forthcoming. We can see
that, if $\alpha$ is near zero, selling yields higher welfare, but if it is near one (and $v$ is above $9/8$) leasing will do so. So for some $\hat{\alpha}$ between zero and one, the expected welfare from leasing and selling are the same, and for higher probabilities of innovating, leasing will yield higher expected welfare. This value of $\alpha$ will satisfy equation 6.

$$
\hat{\alpha} = \frac{\bar{W}_F - W_{rF}}{W_{rS} - W_S + W_{rF} - W_F}
$$

(6)

The region where, ex ante, welfare from leasing exceeds that from selling is shaded in Figure 5. We can then prove the following.

**Proposition 5.** If $v \leq 9/8$ ex ante welfare will be higher under selling than leasing; if $v > 9/8$, ex ante leasing welfare will be higher than selling when $\alpha$ is greater than $\hat{\alpha}$, and lower otherwise.

Welfare in the product market is always higher under selling than leasing when there is no innovation, but the reverse is true when the monopolist has developed a large enough innovation. Consumer surplus is sometimes lower under leasing because of the lessor’s higher price and restricted output, but a lessor will appropriate enough profit that welfare increases for a high new quality.

We can also establish that leasing the good will give the monopolist a stronger incentive to invest in innovation than selling. Because there is no competition from a used-good market, a lessor appropriates more of the surplus from innovating, and is willing to invest more to realize an innovation; see Lemma 11 in the appendix.

Then, similarly to above, we can compare welfare net of investment across a range of fixed costs of developing a new version. Let $f_r$ be a lessor’s maximum willingness to pay to innovate with probability $\alpha$, and as above $\bar{f}$ is the most a seller will invest to innovate with probability $\alpha$ under incomplete information. If the fixed cost $f(v)$ is below $\bar{f}$, a seller and a lessor will invest $f(v)$, and Proposition 5 gives us the appropriate welfare comparison. If instead the fixed cost is above $f_r$, neither a seller nor a lessor will invest, and we can apply Lemma 2 to show welfare will be higher under selling. For intermediate levels of the fixed cost, a lessor, but not a seller, would try to innovate.
Suppose welfare under leasing is at least as high in this case as selling, as below in inequality 7.

\[ \alpha W_{rS} + (1 - \alpha)W_{rF} - f \geq \tilde{W}_F \]  

Let \( \alpha_r \) be the value of \( \alpha \) for which this holds with equality, i.e., \( \alpha_r = (\tilde{W}_F + f - W_{rF})/(W_{rS} - W_{rF}) \). So inequality 7 holds when the probability of success is at least \( \alpha_r \). Then we have shown the proposition below.

**Proposition 6.** If the fixed cost of innovation is below \( \tilde{f} \), expected welfare net of innovation costs is higher under leasing than selling if \( v > 9/8 \) and the probability of success is at least \( \hat{\alpha} \), and lower otherwise. If the fixed cost of innovation is between \( \tilde{f} \) and \( f_r \), expected welfare net of the fixed cost will be higher under leasing than selling welfare when the probability of success is above \( \alpha_r \), and equal or lower otherwise. If the fixed cost of innovation is above \( f_r \), welfare is higher under selling than leasing.

In addition to letting a monopolist avoid competition with his own future output, leasing keeps consumers’ uncertainty about an improved good from altering current demand. The superiority of leasing for the monopolist is common in this literature. For high levels of new quality, leasing will also be welfare-improving, compared to selling to consumers with incomplete information.

Of course, output is sold in many durable goods markets, possibly because of moral hazard a renter faces in using or abusing the good. Though I do not model this moral hazard problem, the prevalence of selling makes it reasonable to examine the case of a durables seller, as I have above.

V. Discussion

Prices can signal innovation when consumers are uncertain about the firm’s success or failure in bringing a new product to market. A seller’s price will fully reveal his success or failure to innovate for ‘intuitive’ beliefs. A successful innovator is never worse off than in full information, and a failed innovator is always worse off than in
full information.

In this model, the consumers’ information problem intensifies a failed monopolist’s Coasian problem. Compared to the full-information case, a steep discount is required to keep consumers from delaying purchase because of the possible introduction of a new version of the good. Not only does innovation help a monopolist partially escape the Coasian problem, but failure to innovate makes the problem worse. Consumers are better off for initially lacking information about a new good. A policy trying to alleviate their information ‘problem’ by requiring firms to disclose any new version would reduce welfare when prices are informative.

This throws light on product development investment decisions for durable goods. Failing to develop a new good will cost the firm not only the prospective profits from the improved good, but will incur distortions of its price and profits in order to convince consumers not to delay purchase in expectation of a new good that is not coming. Not only would a policy requiring disclosure increase prices and reduce welfare when prices are informative, but it would also remove this added incentive of a firm to succeed in marketing new versions.

This model has abstracted from competition and retail distribution networks; it has focused on consumers’ uncertainty and the seller’s ability to inform them about new versions of a good. Even inexperienced consumers, who are unaware of a seller’s reputation or prior investment activity, can be informed by this signal. The prediction of a high price from an innovator and low from a firm failing to innovate is not one about a sale to reduce inventory. Rather, one might expect to see a larger difference in prices between innovative and non-innovative firms in industries where consumers are not well informed about new versions than where they are.

For instance, a video game publisher may charge a lower price on a current game if development of a sequel has stalled than when it proceeds apace. In the video-game industry, pre-announcements, delays and cancelations are common, and may make announcements non-credible cheap talk.26 A firm that has failed to develop a pre-

26See, for instance, Chris Kohler, “Hi, I’m the Gaming Industry, and I’m addicted to vaporware,” Wired,
announced sequel will have reason to sell the current version at a low price, to induce purchase now. In 2006, delays in the marketing of Sony’s PS3 delayed the release of every game for the PS3. This resulted in low prices on PS2 games with planned sequels on the PS3: Kris Graft writes that in the wake of the PS3 delays, “[w]e’ve already seen substantial price-cuts from EA, as the current-gen versions of The Godfather, Black and Fight Night Round 3 all debuted or will debut at $40, a 20 percent cut from the normal $50 price tag.” Indeed, Kraft and Kwak write that “[t]he biggest risk to the video game market is not the direct revenue impact from a longer rollout of PS3 hardware...but the resulting negative effect on the PS2 software market...any further PS3 delays will more likely hurt publishers’ revenue on PS2 than on PS3.” Because an announcement may not convince consumers that the new version will not arrive, the developer will need to set a low price to stop consumers from waiting for the sequel.

This model admits of extension in a few directions. The monopolist in this setting might seek to deter entry into its market, requiring signaling to multiple audiences. We stipulated that all investment in research and development occur before any sales in the product market, so the seller does not experience any dynamic inconsistency in his choice of investment. It would be interesting to extend this framework when investment occurs after first-period sales, as in Waldman (1996), or at multiple dates over time, as in Utaka (2011). Segments of the business press are devoted to informing consumers about the future release and the quality of many consumer durable goods. But, in this model, consumers should not seek more information about future goods. It may be worth exploring whether consumers facing multiple sellers benefit more from such information on the goods available, or lose through being informed as is the case here.


27 See Kris Graft, “Current-Gen Woes,” Next-Gen, March 7, 2006, retrieved from https://web.archive.org/web/20071021133526/http://www.next-gen.biz/index.php?option=com_content&task=view&id=2428&Itemid=2. Though the article suggests an inventory or price discrimination motive for these low prices, the context commends an explanation like that explored here: consumers would delay purchases in favor of the sequel, and developers must substantially lower the price to prevent this delay.


29 I thank an anonymous referee for this suggestion.
Appendix

A. Proof of Lemma 1

The full-information prices are always ordered \( p^S_1 < p^F_1 \), and the full-information profit of type \( S \) exceeds that of type \( F \).

Proof. We can simplify \( p^S \) to \( \frac{(4 + \delta)(1 + \delta)}{4 + \delta} - \frac{\delta}{(\delta + 4)v + 3\delta \Delta v} \). Suppose to the contrary that

\[
\frac{(2 + \delta)^2}{4 + \delta} > \frac{(4 + \delta)(1 + \delta)}{4 + \delta} - \frac{\delta}{(\delta + 4)v + 3\delta \Delta v}
\]

and therefore, \((4 + \delta)^{-1} < ((\delta + 4)v + 3\delta \Delta v)^{-1} \rightarrow (\delta + 4)v + 3\delta \Delta v < (\delta + 4)\), and since \( \Delta v \equiv v - 1 > 0 \), this means that \(-(4 + \delta) > 3\delta\), requiring that \( \delta < -1 \), in contradiction of hypothesis.

To see that the innovator’s full-information profit is greater than a non-innovators, note that as the improvement of the good becomes vanishingly small—as \( v \) tends to one—the innovator’s profit will tend to that of a non-innovator. Those profits are

\[
\Pi^S = \frac{v(4 + (4 + \delta)\delta v + 3\delta^2(v - 1))}{4((4 + \delta)v + 3\delta(v - 1))} \quad \text{and} \quad \Pi^F = \frac{(2 + \delta)^2}{4(4 + \delta)}.
\]

As the quality of the potential new good grows, the innovator’s profit will grow; or, as \( v \) tends to one, it will tend to the non innovator’s profit. So for any quality improvement, an innovator’s full-information profit will exceed that of the non-innovating type of the firm. \( \square \)

B. Proof of Proposition 1

There is no equilibrium in which a firm of type \( S \) charges the full-information price \( p^S \) and a firm of type \( F \) charges \( p^F \) in the first period.

Proof. Suppose that these prices do support a separating equilibrium. That is, consumers who see a price of \( p^S \) believe the firm is of type \( S \) with probability near one, and of \( p^F \), type \( S \) with near zero probability. Then if \( S \) were to imitate \( F \)’s
price, consumers would react as though the firm were of type \( F \), so that \( S \)'s profit is 
\[
\bar{p}^F(1 - \theta^F) + (\delta/4v)(\theta^F + v - 1)^2,
\]
where as above \( \theta^F = \bar{p}^F/(1 + \delta/2) \). Separation requires that this be no more than \( S \)'s full-information profit; that is,
\[
\frac{4\delta + \delta(\delta + 4)^2v^2 + 16v}{4(\delta + 4)^2v} \leq \frac{v(-3\delta^2 + 4(\delta + 1)\delta v + 4)}{16(\delta + 1)v - 12\delta}
\]
which reduces to the condition that \( \frac{\delta(v-1)(\delta(v-3)-8v)}{(\delta+4)v((\delta(v-3)+4)v)} \geq 0 \). Since \( 0 < \delta < 1 \) and \( 1 < v \), the denominator is positive, and we are left with \( \delta(v-1)(\delta(v-3)-8v) \geq 0 \), which again can only be true if \( -3\delta \geq (8 - \delta)v \). For the same parameter restrictions, the right hand side is positive and the left negative, a contradiction. Then \( S \)'s profit when imitating \( F \) with a price of \( \bar{p}^F \) exceeds the full-information profit \( \Pi^S \). Therefore, it cannot be an equilibrium for each type to set its full-information price, since \( S \) has a beneficial deviation. \( \Box \)

C. Proof of Proposition 2

In any separating equilibrium satisfying the Intuitive Criterion, \( S \) will set the full-information price \( \bar{p}^S \) and \( F \) set \( \bar{p} \).

Proof. The full-information price \( \bar{p}_1^S \) is the unique maximizer of \( \Pi^S \) when consumers believe the firm has an innovation with probability 1. In a separating equilibrium in which \( S \) sets any other price, deviating to \( \bar{p}_1^S \) is not equilibrium dominated, since \( S \) could conceivably get \( \Pi^S(\bar{p}_1^S, 0) > \Pi^S(\bar{p}_1^S, 1) > \Pi^S(p, 1) \) \( \forall p \neq \bar{p}^S \); the worst \( S \) can do by setting this price is exactly \( \Pi^S \), which is better than any other separating equilibrium profit.

We make use of Lemma 4 for cases where \( F \) sets a price above \( \bar{p}^F \) and note that, by construction, prices for \( F \) in between \( \bar{p} \) and \( \bar{p}^F \) will not sustain separation. Suppose instead the failed innovator sets a price \( p \) which sustains separation but is below \( \bar{p} \). The failed innovator’s profit is, from Lemma 5, strictly increasing for prices lower than \( \bar{p}^F \). Therefore, if \( F \) sent the out-of-equilibrium signal \( p' = \bar{p} - \varepsilon \), with \( p' - p > \varepsilon > 0 \), consumers would believe the firm to be of type \( F \) for sure (since the best \( S \) can do here
is $\Pi^S(p', 0) < \Pi^S$, and $F$ would benefit from this deviation, so such an equilibrium does not pass the Intuitive Criterion.

Since all rivals fail the Intuitive Criterion, $p^S$ and $\bar{p}$ are the only prices which sustain an intuitive separating equilibrium. \hfill \Box

**Lemma 4.** In any separating equilibrium which satisfies the Intuitive Criterion $F$ will set a first-period price $p < \bar{p}_1^F$.

**Proof.** $F$ can use prices $b$ or $q$ to signal his or her type, where $b < \bar{p}^F < q$ and $\Pi^S(b, 0) = \Pi^S(q, 0) \leq \Pi^S$. Note that because the profit $S$ would get from imitating $F$ is quadratic in prices, we can rewrite it in terms of squared deviations from its maximizer, $p^S(0)$. Then $b$ and $q$ so defined will be equidistant from $p^S(0)$, in that $b = p^S(0) - d$ and $q = p^S(0) + d$ for some distance $d$; but $F$ will always have higher profit at the lower price. Take a separating equilibrium where $F$ sets $q$; then $F$ could set a price $p = b - \varepsilon$, which is equilibrium dominated for $S$, so the deviation comes surely from $F$, and for small $\varepsilon$, $\Pi^F(p, 0) > \Pi^F(q, 0)$. Therefore, a separating equilibrium in which $F$ sets a price above $\bar{p}_1^F$ admits of a beneficial deviation which consumers will credit as being by a type-$F$ firm, and therefore does not meet the Intuitive Criterion.

To see that $\Pi^F(b, 0) > \Pi^F(q, 0)$, suppose to the contrary that $\Pi^F(b, 0) \leq \Pi^F(q, 0)$, which means that

$$b(1 - \frac{b}{1 + \delta/2}) + \frac{\delta}{4}(\frac{b}{1 + \delta/2})^2 \leq q(1 - \frac{q}{1 + \delta/2}) + \frac{\delta}{4}(\frac{q}{1 + \delta/2})^2$$

and therefore $\frac{(q - b)(q + b)}{(2 + \delta)^2} (-4 - \delta) \geq b - q$ and $q + b \leq \frac{(2 + \delta)^2}{4 + \delta} = 2\bar{p}^F$. Since $b$ and $q$ are equidistant from $p^S(0)$, $b + q$ is simply $2p^S(0)$. That is,

$$p^S(0) = \frac{(2 + \delta)(2v + 2v\delta - \delta)}{4v + 2v\delta - \delta} \leq \frac{(2 + \delta)^2}{2(4 + \delta)}.$$

This implies $4v + 6v\delta - 3\delta + v\delta^2 - \frac{\delta^2}{2} \leq 0$, and in turn that

$$v \leq \frac{\delta(3 + \delta/2)}{4 + 6\delta + \delta^2} = \frac{\frac{1}{2}(6\delta + \delta^2)}{4 + 6\delta + \delta^2} < \frac{\frac{1}{2}(6\delta + \delta^2)}{6\delta + \delta^2} = \frac{1}{2}.$$
Since we have restricted the new quality to be strictly greater than 1, this is a contradiction. Therefore $\Pi^F(b, 0) > \Pi^F(q, 0)$ and $F$ will separate with a lower price than in full-information in any separating equilibrium which passes the Intuitive Criterion. □

**Lemma 5.** The profit of $F$ in a separating equilibrium is strictly increasing for prices below $p^F$.

*Proof.* $F$’s profit in a separating equilibrium will be $\Pi^F(p, 0) = p(1 - \frac{p}{1+\delta/2}) + \frac{\delta}{4}(p/(1+\delta/2))^2$. This has a partial derivative of $\frac{\delta p}{2(\frac{\delta}{2}+1)^2} - \frac{2p}{\frac{\delta}{2}+1} + 1$ which has a root at $p^F = (2 + \delta)^2/(4 + \delta)$ and will be strictly positive for lower prices. Therefore $F$’s profit in a separating equilibrium will be increasing in $p$ as long as the first-period price is below the full-information price. □

In addition, we can prove that the above equilibrium will always exist. By construction $S$ has no incentive to imitate $F$, but $F$ will also never have an incentive to imitate $S$’s first-period price.

**Lemma 6.** The failed type’s profit when setting $\bar{p}$ exceeds that from any deviation for which consumers believe the firm is of type $S$ for sure.

*Proof.* Define the surface $A(\delta, v)$ by $\Pi^F(p^F(A), A) = \Pi^F(\bar{p}, 0)$. From Lemma 7 we have that $\Pi^F(p^F(A), A) > \Pi^F(p^F(1), 1)$, and because $p^F(1)$ maximizes $\Pi^F(p, 1)$, we know that $\Pi^F(p^F(1), 1) \geq \Pi^F(p, 1)$ and therefore $\Pi^F(\bar{p}, 0) > \Pi^F(p, 1)$ for any first-period price $p$. □

In particular, the profit when setting $\bar{p}$ will exceed that from imitating $S$ in the first period, so this separating equilibrium will always exist.

**D. Proof of Proposition 3**

No pooling equilibrium survives the Intuitive Criterion.

*Proof.* We must show that for each pooling equilibrium there is a deviation price that a seller of type $S$ would never set, and that would benefit a seller of type $F$, given this.
It will be useful to restrict the range of prices we consider to \( p \in (\bar{p}, \bar{p}^+) \) with Lemma 9.

Fix a pooling equilibrium at price \( p \) when the prior probability the seller has a new version is \( \alpha \). Then we can find two prices \( q(p, \alpha) \) for which, even at best, a seller of type \( S \) is no better off than in equilibrium. These will satisfy \( \Pi^S(q, 0) = \Pi^S(p, \alpha) \). By an argument similar to that in Lemma 4, the higher of these prices will not always be a beneficial deviation for \( F \) and will no longer concern us. The lower of the two is given below.

\[
2 \left( \delta^2 + 3\delta + 2 \right) v - \delta^2 - 2\delta - \sqrt{(\delta + 2)^2v (4\delta \Pi^S(p, \alpha) - (\delta + 2)v(\delta + 8\Pi^S(p, \alpha) - 2) + 2\delta(\delta + 2)v^2)} \leq 4(\delta + 2)v - 2\delta
\]

This price \( q \), which by Lemma 8 is below \( p \), is a downward deviation from \( S \)'s best price given \( \alpha \), which is below \( \bar{p}^S \), per Lemma 7 (iii). Since a seller of type \( S \) is no better off setting \( q \) than in equilibrium even at best, and per Lemma 7 (iii) \( S \)'s profit is increasing in price when the price is below \( \bar{p}^S \), a seller of type \( S \) would be strictly worse off than in equilibrium setting a price below \( q \). Then if consumers see a price just below \( q(p, \alpha) \), they will believe it comes from a seller of type \( F \) with probability near one, so a seller of type \( F \) would gain \( \Pi^F(q, 0) \) from this deviation.

It remains to show that this profit is better than \( F \)'s profit in equilibrium. By construction, \( S \) is indifferent between the equilibrium profit of \( \Pi^S(p, \alpha) \) and the best profit from setting \( q \) below \( p \), that is, \( q(1 - \theta(q, 0)) + \delta(v - 1 + \theta(q, 0))^2/4v = \Pi^S(p, \alpha) \). Suppose to the contrary that, at best \( F \) is no better off deviating to this price from a pooling equilibrium : \( q(1 - \theta(q, 0)) + \delta(\theta(q, 0))^2/4 \leq \Pi^F(p, \alpha) \). We will write \( \theta_q \) for \( \theta(q, 0) \) and \( \theta_p \) for \( \theta(p, \alpha) \). Then, we can also write

\[
\Pi^S(p, \alpha) - \delta(v - 1 + \theta_q)^2/4v + \delta(\theta_q)^2/4 \leq \Pi^F(p, \alpha)
\]

or \( \Pi^S(p, \alpha) - \Pi^F(p, \alpha) \leq \delta(v - 1 + \theta_q)^2/4v - \delta(\theta_q)^2/4 \), and then plug in the profit functions on the left hand side to get \( \delta(v - 1 + \theta_p)^2/4v - \delta(\theta_p)^2/4 \leq \delta(v - 1 + \theta_q)^2/4v - \delta(\theta_q)^2/4 \).
\[\delta(\theta_q)^2/4 \text{ and thus that } 2\theta_p - \theta_p^2 \leq 2\theta_q - \theta_q^2 \text{ and} \]

\[(\theta_q + \theta_p)(\theta_q - \theta_p) \leq 2(\theta_q - \theta_p).\]

Since \(\theta\), consumers' marginal value for quality, lies between zero and one, it must be that \(\theta_q \geq \theta_p\), that is, \(q/(1 + \delta/2) \geq (p + \alpha \delta(v - 1)/2v)(1 + \delta/2 + \alpha \delta(v - 1)/2v)\) and therefore

\[(1 + \delta/2)(q - p) \geq (1 + \delta/2 - q)\alpha \delta(v - 1)/2v.\]

The right hand side is positive, since \(1 + \delta/2\) is the full-information choke price for \(F\), above \(p\) and \(q\). But, since \(q\) is below \(p\), the left hand side is negative. This is a contradiction, so it must be that \(F\) is strictly better off setting \(q\) — and by continuity, a price near \(q\) — than setting a pooling price \(p\).

\[\square\]

**Lemma 7.** (i) Both types' profits are strictly decreasing in the posterior belief that the firm is of type \(S\) for prices in between \(\frac{\delta(v - 1)(\delta \mu + \delta + 2)}{2(\delta \mu + \delta + 2v - \delta (\mu + 1))} \) and \(1 + \delta/2\).

(ii) Both types' maximum profits at a given posterior are decreasing in the posterior.

(iii) Both types' profits are strictly increasing for prices below \(p^F\).

**Proof.** (i) The profit of a non-innovating firm is

\[p \left(1 - \frac{p + \mu \delta(v - 1)/2v}{1 + \delta/2 + \mu \delta(v - 1)/2v}\right) + \frac{\delta}{4} \left(\frac{p + \mu \delta(v - 1)/2v}{1 + \delta/2 + \mu \delta(v - 1)/2v}\right)^2,\]

with a partial derivative in \(\mu\) of

\[
\frac{\delta^2(v - 1) \left(p + \frac{\delta \mu(v - 1)}{2v}\right)}{4v \left(\frac{\delta}{2} + \frac{\delta \mu(v - 1)}{2v} + 1\right)^2} - \frac{\delta^2(v - 1) \left(p + \frac{\delta \mu(v - 1)}{2v}\right)^2}{4v \left(\frac{\delta}{2} + \frac{\delta \mu(v - 1)}{2v} + 1\right)^3}
\]

\[+ p \left(\frac{\delta(v - 1) \left(p + \frac{\delta \mu(v - 1)}{2v}\right)}{2v \left(\frac{\delta}{2} + \frac{\delta \mu(v - 1)}{2v} + 1\right)^2} - \frac{\delta(v - 1)}{2v \left(\frac{\delta}{2} + \frac{\delta \mu(v - 1)}{2v} + 1\right)}\right)\]
This will be strictly negative when \( \frac{\delta^2 \mu (v-1)}{2[(2+\delta)v + \delta \mu (v-1)]} < p < \frac{\delta + 2}{2} \); note that \( \frac{\delta^2 \mu (v-1)}{2[(2+\delta)v + \delta \mu (v-1)]} < \tilde{p} \) and \( \tilde{p}^+ < \frac{2 + \delta}{2} \).

The profit of an innovating firm is

\[
p \left( 1 - \frac{p + \mu \delta (v-1)/2v}{1 + \delta/2 + \mu \delta (v-1)/2v} \right) + \frac{\delta}{4v} \left( v - 1 + \frac{p + \mu \delta (v-1)/2v}{1 + \delta/2 + \mu \delta (v-1)/2v} \right)^2,
\]

with a partial derivative in \( \mu \) of

\[
\begin{align*}
&\frac{p}{2v} \left( \frac{\delta (v-1) \left( p + \frac{\delta \mu (v-1)}{2v} \right)}{\left( \frac{\delta}{2} + \frac{\delta \mu (v-1)}{2v} + 1 \right)^2} - \frac{\delta (v-1)}{2v \left( \frac{\delta}{2} + \frac{\delta \mu (v-1)}{2v} + 1 \right)} \right) \\
&\quad + \frac{\delta}{2} \left( \frac{\delta (v-1)}{2v \left( \frac{\delta}{2} + \frac{\delta \mu (v-1)}{2v} + 1 \right)} - \frac{\delta (v-1) \left( p + \frac{\delta \mu (v-1)}{2v} \right)}{2v \left( \frac{\delta}{2} + \frac{\delta \mu (v-1)}{2v} + 1 \right)^2} \right) \left( \frac{p + \frac{\delta \mu (v-1)}{2v}}{2v + \frac{\delta \mu (v-1)}{2v} + 1} + v - 1 \right)
\end{align*}
\]

This will be strictly negative when

\[
\frac{(2 + \delta (1 + \mu)) (\delta (v-1))}{2(2v + \delta (1 + \mu) (v-1))} < p < \frac{2 + \delta}{2}
\]

which interval includes \([\tilde{p}, \tilde{p}^+]\).

(ii) Both types’ maximum profits at a given posterior are decreasing in the posterior. Along with this comparison of profits from different posteriors at a given price, it will be useful to compare the maximum profits for a given posterior.

Let \( p^t(\mu) \) be the best price for type \( t \) when consumers have posterior belief \( \mu \) the seller is of type \( S \). The maximized profit for a type-\( F \) firm when consumers believe it is of type \( S \) with probability \( \mu \) is \( \Pi^F(p^F(\mu), \mu) = \frac{(2 + \delta)^2 v + 2 \mu \delta^2 (v-1)}{4(4 + \delta)v + 2 \delta \mu (v-1)} \).

The partial with respect to \( \mu \) is \( -\frac{2 \delta \mu (v-1) v}{(4 + \delta)v^2 + 2 \delta \mu(v-1)} < 0 \). The maximized profit of a type-\( S \) firm when consumers believe it is of type \( S \) with probability \( \mu \) is \( \Pi^S(p^S(\mu), \mu) = \frac{v(2 \mu (v-1) + 2v - 1) + 4 \delta v + 4}{8v(2 \mu + 1) \delta} \) and its partial with respect to \( \mu \) is \( -\frac{2 \delta \mu (v-1) v}{(4 + \delta)v^2 + 2 \delta \mu (v-1)} < 0 \).

(iii) Both types’ profits are strictly increasing for prices below \( p^F \). The profit functions of each type are as in the full-information benchmark, with the difference that
\[ \theta = \frac{p + \mu \delta(v-2)/2v}{1 + \delta/2 + \mu \delta(v-2)/2v}, \text{ so that} \]

\[ \frac{\partial \Pi^F}{\partial p} = 1 - \frac{p}{\mu \delta(v-1)/2v} + \frac{\delta}{2} + 1 - \frac{\mu \delta(v-1)/2v + p}{\mu \delta(v-1)/2v + \delta/2 + 1} + \frac{\delta \left( \frac{\mu \delta(v-1)/2v + p}{\mu \delta(v-1)/2v + \delta/2 + 1} \right)^2}{2}, \]

and

\[ \frac{\partial \Pi^S}{\partial p} = 1 - \frac{p}{\mu \delta(v-1)/2v} + \frac{\delta}{2} + 1 - \frac{\mu \delta(v-1)/2v + p}{\mu \delta(v-1)/2v + \delta/2 + 1} + \frac{\delta \left( \frac{\mu \delta(v-1)/2v + p}{\mu \delta(v-1)/2v + \delta/2 + 1} + v - 1 \right)}{2v \left( \frac{\mu \delta(v-1)/2v + \delta/2 + 1}{2} \right)^2}. \]

Solving these for \( p \) will yield 

\[ p^F(\mu) = \frac{(2 + \delta)^2 v + 2 \mu \delta(1 + \delta)(v - 1)}{2(4 + \delta)v + 2 \mu \delta(v - 1)}, \]

and

\[ p^S(\mu) = \frac{1}{2} \left( 1 + \delta - \frac{\delta}{(4 + \delta)v + 2 \delta \mu(v - 1) + \delta(v - 1)} \right), \]

the respective maximizers of the types’ profits given posterior belief \( \mu \). Note that these maximizers are increasing in \( \mu \). For \( \mu = 0 \), the first of these partials will have its only root at \( p^F \), and will be positive for any lower price so \( \Pi^F(p, \mu) \) is increasing for prices below \( p^F \). The second partial behaves in the same way for \( \mu = 1 \) and \( p^S \). Since \( p^F_1 < p^S_1 \), \( \Pi^S \) is increasing for prices below \( p^F \).

\[ \square \]

**Lemma 8.** The lower of the prices \( q \) which sets \( \Pi^S(q, 0) \) equal to \( \Pi^S(p, \alpha) \) will be less than \( p \).

**Proof.** We will first introduce some notation. Let \( p^S(\alpha) \) be the maximizer of \( S \)'s profit given \( \alpha \), so that \( \Pi^S(p^S(\alpha), \alpha) \) is \( S \)'s best profit when consumers believe he is of type \( S \) with probability \( \alpha \). Recall that, because the profit of the seller is quadratic in price, we can write it in “vertex form,” as its maximum less an amount proportional to the squared deviation from the maximizer of the seller’s profit, as in equation 8 below.

\[ \Pi^S(p, \alpha) = \Pi^S(p^S(\alpha), \alpha) - \lambda_\alpha(p^S(\alpha) - p)^2 \]  

(8)
A bit of arithmetic will show that \( \lambda_\alpha \) is 
\[
\frac{v(2v(\alpha \delta + \delta + 2)-(2\alpha + 1)\delta)}{(\alpha - v(\delta + \delta + 2))^2}.
\]
We can define \( q \) in this way, as well, as the lower solution to \( \Pi^S(p, \alpha) = \Pi^S(p^S(0), 0) - \lambda_0(p^S(0) - q)^2 \), so
\[
q = p^S(0) - \sqrt{[\Pi^S(p^S(0), 0) - \Pi^S(p, \alpha)]/\lambda_0}.
\]

Suppose instead that the pooling price \( p \) is below \( q \). Note that, since \( q \) is \( p^S(0) - S \)'s best price when \( \alpha \) is zero—minus a radical term, we are also assuming that \( p \) is below \( p^S(0) \). Then \( p \leq q \) means that \( \lambda_0(p - p^S(0))^2 \geq \Pi^S(p^S(0), 0) - \Pi^S(p, \alpha) \), or \( \Pi^S(p, \alpha) \geq \Pi^S(p^S(0), 0) - \lambda_0(p - p^S(0))^2 = \Pi^S(p, 0) \). But, by Lemma 7 (i), we know that this must be false, as \( \Pi^S(p, 0) > \Pi^S(p, \alpha) \) for positive \( \alpha \).\(^{30}\) So the price \( q(p, \alpha) \) must be below the price \( p \) from which (at best) it makes a seller of type \( S \) indifferent to deviating.

\[\Box\]

**Lemma 9.** In any pooling equilibrium satisfying the Intuitive Criterion, \( S \)'s profit \( \Pi^S(p, \alpha) \geq \Pi^S \).

**Proof.** Suppose to the contrary that a pooling equilibrium at \( p^* \) is such that \( \Pi^S(p^*, \alpha) < \Pi^S \). Then if \( S \) were to deviate to \( p^S \), the best \( S \) can do is \( \Pi^S(p^S, 0) > \Pi^S > \Pi^S(p^*, \alpha) \), so this deviation is not equilibrium dominated. The worst \( S \) can so is \( \Pi^S(p^S, 1) = \Pi^S(p^S, 0) > \Pi^S > \Pi^S(p^*, \alpha) \), so that \( S \) can always benefit from this deviation. Such an equilibrium does not pass the Intuitive Criterion. Since \( \Pi^S(p^S, 1) = \Pi^S(p^S, 0) = \Pi^S(p^S, 0) \), only prices in between \( \tilde{p} \) and \( \tilde{p}^+ \) can support a pooling equilibrium with higher profit than full information.\[\Box\]

**E. Proof of Proposition 5**

If \( v \leq 9/8 \) welfare will be higher under selling than leasing; if \( v > 9/8 \), leasing welfare will be higher than selling when \( \alpha \) is greater than \( \hat{\alpha} \), and lower otherwise.

**Proof.** If quality is below 9/8, we recall Lemmas 2 and 3. Welfare is lower under leasing than selling whether innovation fails or succeeds.\(^{30}\)If the pooling price is outside the range where that lemma applies, we can invoke Lemma 9, instead.
If instead the new quality is high, we take account of $\alpha$. If it is below the level $\hat{\alpha}$ given in equation 6, then the negative first term and positive last term of expression 5 are weighted so the whole is positive. That is, the expected welfare from leasing will be lower than that from selling. If instead $\alpha$ is higher, the higher surplus under leasing when innovation succeeds will more than balance the loss of surplus when innovation fails, and expected surplus from leasing will be greater than that from selling. 

Proof of Lemma 2. If innovation has failed, welfare is higher under selling than leasing.

Proof. When innovation has failed, we can compare this welfare to full-information welfare when the realized type is $F$ from above, $W_F = \frac{1}{2}(1 - \bar{\theta}_1^2) + \frac{\delta}{2}(1 - \bar{\theta}_2^2)$. Recall that $\bar{\theta}_2^F = \bar{\theta}_1^F / 2$ and $\bar{\theta}_1^F = (2 + \delta) / (4 + \delta)$. Then the difference of welfare between full-information selling and leasing when there is no new version is

$$\frac{2\delta^2 + 11\delta + 8}{8(\delta + 4)} > 0,$$

so that full-information selling welfare is always higher then leasing welfare when innovation has failed. We showed above in section III.D. that welfare is higher in the separating equilibrium than in full information when $F$ is the realized type. It follows that this is also greater than leasing welfare.

Proof of Lemma 3. If innovation has succeeded, welfare is higher under selling than leasing when $v < 9/8$, and lower otherwise.

Proof. Recall from section III.D. that welfare when a seller has a new version is

$$W_S = \frac{v(3\delta^2(4v - 3) + 4\delta(3v + 1) + 12)}{8(4(\delta + 1)v - 3\delta)},$$

which we compare to total leasing welfare of $W_{rS} = (3/8)(1 + \delta v)$. The difference of these will be $\delta(9 - 8v) / 8 [4(\delta + 1)v - 3\delta]$, which is negative for any $v$ greater than $9/8$. For a lower $v > 1$, there will be higher welfare for selling than leasing. 

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Lemma 10. The producer will not rent the original good in the second period.

Proof. We allow the seller to sell both versions in the second period. Let the rate for the new version be \( r_v \) and the old \( r_o \). A consumer who is just indifferent between renting the new and old will have a marginal value for quality \( \theta_v \) which satisfies \( \theta_v v - r_v = \theta_v - r_o \), or \( \theta_v = (r_v - r_o)/(v - 1) \) so that all \( 1 - \theta_v \) consumers with higher \( \theta \) rent the new good. The marginal renter of the old good will have a marginal value for quality \( \theta_o \) which satisfies \( \theta_o = r_o \), so he or she is just indifferent to renting the old good, and the measure of renters of the old good will be \( \theta_v - \theta_o \) or simply \( \theta_v - r_o \). The producer then has the following problem in the second period.

\[
\max_{r_0, r_v} r_v (1 - \theta_v) + r_o (\theta_v - r_o),
\]

This will be solved by \( r_0 = 1/2 \) and \( r_v = v/2 \). Plugging these in, we find that \( \theta_v = 1/2 \), and therefore that, in equilibrium, no consumers rent the old version, and the producer charges the same rental rate on the new version as when the producer only rents the new in the second period.

Lemma 11. A monopoly lessor will have a stronger incentive to invest in innovation than a seller.

Proof. The expected profit from renting will be \( \alpha \Pi^S + (1 - \alpha) \Pi^F \), and the most a lessor will pay to have a chance \( \alpha \) of marketing a new quality \( v \) will be \( \alpha (\Pi^S - \Pi^F) = \alpha \delta(v - 1)/4 \). We compare this willingness-to-pay to that when selling in the separating equilibrium, \( \alpha (\Pi^S - \Pi^F(\tilde{p}, 0)) \). Since we defined \( \tilde{p} \) so that \( \Pi^S(\tilde{p}, 0) = \Pi^S \), we can write this as \( \alpha (\Pi^S(\tilde{p}, 0) - \Pi^F(\tilde{p}, 0)) \) or \( \alpha [(\delta/4v)(v - 1 + \tilde{\theta})^2 - \delta \tilde{\theta}^2/4] \).

Suppose to the contrary that the expected benefit of investment for a lessor is smaller than that for a seller, that is, \( \delta(v - 1)/4 \leq (\delta/4v)(v - 1 + \tilde{\theta})^2 - \delta \tilde{\theta}^2/4 \). Then we can also say \( 1 \leq (v - 1 + 2\tilde{\theta} - \tilde{\theta}^2)/v \) and finally that \( (1 - \tilde{\theta})^2 \leq 0 \), a contradiction, since \( \tilde{\theta} \) is strictly less than one. Therefore, the marginal benefit of innovation must be strictly greater for a lessor than a seller.
References


Figure 4: Surplus and Deadweight Loss

Figure 5: The region where $\alpha$ is greater than $\hat{\alpha}$, for $\delta = 19/20$. 