Policies to Combat Child Labor: A Dynamic Analysis*

Satya P. Das (Indian Statistical Institute)
Rajat Deb (Southern Methodist University)

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Abstract

This paper analyzes child labor in a fully dynamic model with credit constraints. It considers the long-run and short-run effects of an array of policies like lump-sum subsidy, enrollment subsidy, improvement in primary education and variations in loan market parameters. It is shown that some policies that reduce child labor in the long run may lead to an increase in child labor in the short run. Marginal changes in the borrowing rate or credit limit do not affect the long-run incidence of child labor if the rate of time preference is constant. Implications of variable rate of time preference are also examined.

Address correspondence to
Rajat Deb
Department of Economics
Southern Methodist University
Dallas, Texas 75275
Email: rdeb@mail.smu.edu

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1 Introduction

The practice of child labor plagues many developing economies. According to the recent “Global Report on Child Labor” by ILO (2002), the extent of the problem seems far more serious than was thought earlier. In comparison to an estimate made in 1996 that there are about 250 million children working in developing countries, the current estimate stands at 352 million. The incidence of child labor is highest in the Asia-Pacific region. According to this Report, an alarming number of children are still trapped in the worst forms of child labor, despite “significant progress” in attempts to combat child labor. Making successful policy prescriptions call for a clear understanding of how child labor arises as an economic phenomenon, of different channels through which child labor reducing policies work and of the impact of policies on both child labor and welfare of the (usually poor) households whose children work.

Since the paper by Basu and Van (1998), a substantial theoretical (and empirical\(^1\)) literature has evolved analyzing different causes of child labor and looking at policies for effectively combating the problem, e.g., Balant and Robinson (2000), Ranjan (2001), Dessy (2000), Dessy and Pallage (2001), Dessy and Vencatachellum (2003) and Jafarey and Lahiri (2000, 2002) – to name just a few.

In a multiple-equilibria framework, Basu and Van (1998) show that a ban on child labor may move the labor market from a low adult-wage equilibrium to a “benevolent” high adult-wage equilibrium. Baland and Robinson (2000) demonstrate that even when parents are altruistic, a family may use inefficiently high child labor if it does not make bequests or if the capital market is imperfect. They also show that, a marginal ban on child labor can be Pareto-improving. The impacts of redistributive measures and trade sanctions on the incidence of child labor are analyzed by Ranjan (2001). Jafarey and Lahiri (2000) study the effects of enrollment subsidy and education quality enhancement programs, while Jafarey and Lahiri (2002) examine the effects of trade sanctions. Dessy and Pallage (2001) and Dessy and Vencatachellum (2003) show that child labor may arise as a coordination failure between parents deciding to educate their children and firms deciding to adopt skill-based technology. Parents may decide not to send their children to school if there are not enough firms adopting skill based technology, and, firms may not decide to adopt such technology if there are not enough parents sending their children to school.

In our view, what is lacking in this otherwise impressive and fast-growing literature is a policy analysis in a “fully” dynamic framework of child labor choice. As noted by Basu (1999, p. 110), “One big caveat in the large literature on child labor is the treatment of dynamics. Yet dynamic consequences of child labor are likely to be large since an increase in child labor frequently causes a decline in the acquisition of human capital.” Our paper accepts this basic premise and views child labor as an outcome of intertemporal choice problem facing a poor family choosing between current consumption and future consumption as represented by accumulation of human capital. Child labor is an instrument for shifting consumption from the future to the present. Since the availability of credit impinges critically on intertemporal consumption smoothing, alternative “non-credit market” based policies need to be compared to policies in which the government intervenes to make loans available to families. Against this backdrop, we develop an infinite-horizon dynamic model of child labor incorporating the possibility of government intervention to relax credit constraints. We use a partial equilibrium dynamic analysis to examine how various policies affect the time profile of child labor and family consumption.

In the existing literature, Ranjan (2001) is the only paper, that we are aware of, which has a fully dynamic model of child labor. But, it focuses on the role of income distribution for child labor and his model assumes “no-borrowing”– thus avoiding altogether credit policy issues. All other dynamic models either base themselves on the theoretically unsatisfactory assumption of “warm glow” utility functions, where the child’s education directly enters the adult family member’s utility function (e.g., Galor and Zaira (1993), and Jacoby and Skoufias (1997)) or assume a two-period time horizon, (e.g., Baland and Robinson (1999), and Jafarey and Lahiri (2000, 2002)) and are therefore unsuitable for analyzing long term policy implications or for comparing long term to short term impacts of a policy.

Our paper is complementary to three of the papers mentioned above: Basu and Van (1998), Jafarey and Lahiri (2000) and Ranjan (2001). Like Ranjan (2001), we follow Barro’s (1974) standard approach, and assume that overlapping generations provide an infinite generational link. But, unlike Ranjan (2001), we avoid the issues of income distribution. Instead, as in Jafarey and Lahiri (2000), we examine policies like food-for-education and programs for enhancing the quality of primary education. However, unlike them we use a dynamic partial equilibrium model rather than a two-period general equilibrium model. We also examine other policies, such as poverty alleviation programs, programs for increasing adult wages (via say changes in trade policy or direct labor market interventions), partial and total bans on child labor, as well as policies of changing loan-
market parameters. In other words, we analyze and compare a whole battery of policies including those about credit availability. Our paper relates to Basu and Van (1998) in that there can be multiple equilibria with respect to the incidence of child labor. In their paper multiple equilibria arise via changes in adult and child wages in the labor markets, whereas, in ours, adult and child wages are held constant and multiple equilibria and poverty traps arise from the variability of the intertemporal discount rate.\(^2\)

Why should one want to see child labor reduced? In most models with multiple equilibria, the benevolent equilibrium with no or less child labor may be thought of as being socially better because it increases the well-being of the poor families whose children work. This normative value of reducing child labor is a *derivative* value: it is good because it increases “utility.”\(^3\) However, international agencies such as UNICEF and many developing countries have argued that children have a *right* to be educated, not labored, and child labor, like slavery, is intrinsically evil. Thus, reducing child labor – quite apart from any derivative value – has an *intrinsic* social value. While this argument has not been made in any existing formal model of child labor, it is strongly supported by some of the most recent advances in theoretical normative economics, e.g., Pattanaik, Salles and Suzumura (2004). In our paper the equilibrium paths are utility maximizing (over infinite horizon) for the concerned families. In comparing two such trajectories (in a multiple equilibria situation or across two policy regimes), we assume that child labor is intrinsically bad.\(^4\) Alternative policies are evaluated by looking at the consumption (and hence utility) level of a family as well as the extent of child labor it engages in.

Our analysis proceeds in three steps. We first investigate the impacts of various policies in the absence of any credit market and under the assumption that a family’s discount rate is constant. We then allow for limited access to the credit market.\(^5\) Finally, we consider the implications of variable rate of time discount.

Currently, in order to combat child labor, numerous policy initiatives have been launched by

\(^2\)Our notion of equilibrium is also different. It is that of a dynamic steady state, rather than that of a Nash equilibrium prevalent in “coordination failure” and “general” equilibrium models of child labor.

\(^3\)But this may not represent a Pareto improvement, as for instance, in Basu and Van (1998). An interpersonal comparison of utility is therefore implicit in this normative judgement.

\(^4\)This is somewhat analogous to choosing between two situations where an individual prefers being drunk to being sober when he is drunk and sober to being drunk when sober. A comparison necessarily entails making a value judgement as to the individual’s ‘true’ preference or a judgement about the social value of being sober.

\(^5\)However, our partial equilibrium model is unable to consider an endogenous loan market as in Jafarey and Lahiri (2000, 2002).
the ILO, the World Bank and by the UNICEF. A summary of the various types of policy inter-
ventions can be found in Fallon and Tzannatos (1998). Among those proposed or currently being
implemented are: general poverty reduction measures, policies making basic education available
and compulsory, providing support services for working children, raising public awareness about
the seriousness of the problem, legislating and enforcing child labor laws, trade sanctions, certifi-
cation of products as being child labor free and various lending schemes.\footnote{Fallon and Tzannatos (1998) also list country-specific projects in India and Thailand.} Our theoretical analysis
shows that there are three general determinants of child labor: (a) the family’s valuation of the
(gross) return from education, (b) the opportunity cost of sending a child to school and (c) the
availability and cost of alternative instruments for intertemporal trading. The various policies can
be categorized according to which of the factors (a), (b) or (c) they have as their target. Our
selection of policy instruments is ‘representative’ in that, together, they cover examples that affect
all the three determinants.

Our model offers insights into how different policies work and how they compare against one
another in the short and long run. We evaluate the policies on the basis of their impacts on child
labor, family consumption and the discounted value of the family’s utility. Our general findings
are as follows. First, we are able to identify policies that reduce child labor in the long run but
may not reduce child labor in the short run. Furthermore, policies that reduce child labor in the
long run may in some circumstances increase and in others reduce current consumption of the
families employing child labor. Both these phenomena may impede popular political support for
these policies and thus impinge on the viability of pursuing them. Second, some policies affect the
long-run incidence of child labor only if the discount rate is variable. But there are other policies
that reduce child labor irrespective of whether the discount rate is variable. The latter category of
policies can then be thought of as being more reliable and more potent in combating child labor
than the former, because for this class of policies the variable discount rate produces an additional
channel for impacting child labor. Third and perhaps most surprisingly, (limited) credit market
access belongs to the former category of policies; it affects child labor only if the discount rate is
variable. Furthermore, given that the discount rate is variable, while a decrease in the interest rate
reduces child labor in the long run, an increase in the amount of credit limit increases child labor.
Thus, whereas the existing literature has strongly supported credit availability as an instrument
for reducing child labor, in our model such credit policies are less important; there are other direct
interventions which are potentially more effective.
We now turn to formal analysis.

2 A Base-Line Model

Individuals live for two periods. A representative family has two members: a child and a parent. The family’s consumption (by both members), \( c_t \), depends on the family’s income. No borrowing is allowed. Instantaneous utility, \( u \), is a function of family consumption, \( c_t \). The parent is the sole decision maker. Decision making is forward looking and at time \( t \), the agent maximizes the objective function \( V_t \) given by:

\[
V_t = u(c_t) + \beta V_{t+1}, \beta \in (0,1),
\]

where \( \beta \) is the subjective discount factor, assumed constant. Thus, \( V_t \) consists of (a) the instantaneous utility from consumption in period \( t \) and (b) the discounted value of the child’s objective function when she becomes a parent in the next period. Let \( u' > 0 > u'' \) and let \( u(\cdot) \) satisfy the usual “Inada” conditions.

Such a recursive structure (à la Barro, 1974) implies an infinite chain across generations. By substituting recursively for \( V_{t+1} \) it follows that at \( t = 0 \), the parent maximizes: \( \sum_{t=0}^{\infty} u(c_t)\beta^t \).

At any \( t \) the child and the parent each have a natural labor endowment of one unit in terms of time. The child’s endowment can be used for working \( (l_t) \) or receiving education \( (1-l_t) \). The choice of \( l_t \) rests with the parent. As in Balant and Robinson (2000), there is an education technology described in terms of the consumption good. It is given by \( h(1-l_t) \), \( h' > 0 > h'' \). Let \( w_a \) and \( w_c \) denote respectively the adult (unskilled) wage and the child wage, both defined in terms of the consumption good. These are given and \( w_a \geq w_c > 0 \). Let \( \omega \equiv w_a/w_c \).

It will be convenient to use the notation \( L_t \equiv l_{t-1} \). We can think of \( L_t \) as the child labor ‘embodied’ in the parent at time \( t \). Using this a family’s income in period \( t \) is given by the sum of the parent’s income, \( w_a h(1-L_t) \), and that of the child, \( w_c L_{t+1} \). Thus, the parent’s intertemporal maximization problem is given by max \( \sum_{t=0}^{\infty} u(c_t)\beta^t \), subject to

\[
c_t \leq w_c L_{t+1} + w_a h(1-L_t), \quad t = 0,1,..,\infty
\]

\[
0 \leq L_t \leq 1, \quad t = 1,2,..,\infty.
\]
There is one initial condition: \( L_0 \) is given. Consider the Lagrangian, \( L \), given by:

\[
L = \sum_{t=0}^{\infty} u(c_t) \beta^t + \sum_{t=0}^{\infty} \{ \lambda_t [w_c L_{t+1} + w_a h(1 - L_t) - c_t] \} + \sum_{t=1}^{\infty} \{ \xi_{1t} L_t + \xi_{2t} (1 - L_t) \}.
\]

The choice variables are: \( \{c_t\}_0^{\infty} \) and \( \{L_t\}_1^{\infty} \). The Inada conditions on \( u(\cdot) \) imply \( c_t > 0 \). The first-order condition with respect to \( c_t \) is

\[
\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{\lambda_{t+1}}{\lambda_t}, \quad \lambda_t > 0.
\]

The other first-order conditions are:

\[
\begin{align*}
\lambda_{t-1} w_c - \lambda_t w_a h'(1 - L_t) &= \xi_{2t} - \xi_{1t} \quad (2) \\
\xi_{1t} L_t &= 0, \quad \xi_{1t} \geq 0 \quad (3) \\
\xi_{2t} (1 - L_t) &= 0, \quad \xi_{2t} \geq 0 \quad (4)
\end{align*}
\]

### 2.1 Steady State

Now we will argue that, under reasonable restrictions on the parameters of the model, the steady state exists, is unique and stable. Let us denote steady state values by an asterisk. Eq. (1) reduces to \( \lambda_{t+1}/\lambda_t = \beta \), and we can write eq. (2) as

\[
w_c - \beta w_a h'(1 - L^*) = \frac{\xi_{2t} - \xi_{1t}}{\lambda_{t-1}}.
\]

We will assume that the parameters of the model satisfy the following restrictions:

**INTERIOR-SOLUTION ASSUMPTION:**

\[
\omega h'(0) > \frac{1}{\beta} > \omega h'(1).
\]

Depending on the wage premium of adult labor and the education technology this condition imposes a restriction on \( \beta \). Given this assumption, \( 0 < L^* < 1 \). This implies \( \xi_{1t} = \xi_{2t} = 0 \). Thus, eq. (5) reduces to

\[
7\text{Unlike in finite-period models, there is no need to assume that a parent derives any direct utility from seeing her child educated.}

\[
8\text{Suppose, } L^* = 0. \text{ Then eq. (4) implies } \xi_{2t} = 0 \text{ and eq. (5) reduces to}
\]
\[
\omega h'(1 - L^*) = \frac{1}{\beta},
\] (6)

which determines the long-run equilibrium value of child labor. It is illustrated in Figure 1(a). We see that, since \(h'' < 0\), the steady state must necessarily be unique.

![Figure 1: Steady-State Child Labor and Family Consumption in the Base-Line Model](image)

Child labor arises \((L > 0)\) and persists in the long run when (perhaps because of immediate dire needs caused by poverty) the discount factor of future consumption is sufficiently small relative to the premium on adult labor \((\omega)\) and the marginal productivity of education \((h')\). Under these conditions, a family smooths out consumption over time by using child labor to increase current consumption at the expense of future consumption.

Given \(L^*\), the steady-state level of family consumption is given by:

\[
c^* = w_c L^* + w_a h(1 - L^*). \quad (7)
\]

The overall steady state is illustrated in Figure 1(b). The \(LL\) curve represents (6). Eq. (7) gives the \(CC\) curve.\(^9\)

\[
w_c - \beta w_a h'(1) = -\frac{\xi_{tt}}{\lambda_{t-1}} \Leftrightarrow \frac{1}{\beta} = \omega h'(1) = -\frac{\xi_{tt}}{w_c \beta \lambda_{t-1}}.
\]

However, this cannot hold as long as \(\xi_{tt} \geq 0, \lambda_{t-1} > 0\) and \(1/\beta > \omega h'(1)\). This proves that \(L^* > 0\). Similarly, it can be established that \(L^* < 1\).

Along the \(CC\) curve \(dc/dL = w_c [1 - \omega h'(1 - L)]\). In the neighborhood of the steady state and to the right of \(L^*\), \(1 - \omega h'(.) < 0\), and thus \(dc/dL < 0\), because (a) at the steady state \(\omega h' = 1/\beta > 1\) and (b) \(h'' < 0\). It is possible (but not necessary) that \(dc/dL > 0\) for small values of \(L\).
2.2 Local Dynamics

We will now argue that the steady state is locally saddle-path stable. The family’s budget constraint at time $t$ is given by:

$$L_{t+1} = \frac{1}{w_c} c_t - \omega h(1 - L_t).$$

(8)

Since $1 > L^* > 0$, these inequalities are also satisfied by all values of $L_t$ in a small enough neighborhood of $L^*$. Hence $\xi_{1t} = \xi_{2t} = 0$. We now combine eqs. (1) and (2) and obtain the following Euler equation:

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1}{\omega h'(1 - L_{t+1})}.$$  

(9)

The last two equations govern the local dynamics of $L_t$ and $c_t$. Totally differentiating these equations and evaluating the derivatives at the steady state we get:

$$dL_{t+1} = \frac{1}{\beta} dL_t + \frac{1}{w_c} dc_t; dc_{t+1} = -\frac{\omega h''}{\sigma} dL_t + \left(1 - \frac{h''}{\sigma w_a}\right) dc_t,$$

where $\sigma = -u''/u'$ measures the “curvature of the utility function” and in the presence of risk would be a measure of absolute risk-aversion. It is easy to show that the eigen roots of the system are both positive and only one of them is less than one.\(^\text{10}\) Therefore the system is saddle-path stable. Moreover, the (local) solution of the dynamic path is given by

$$L_t = L^* + (L_0 - L^*) z_2^t; \quad c_t = c^* - w_c \left(1/\beta - z_2\right) (L_0 - L^*) z_2^t,$$

where $z_2 \in (0,1)$ is the stable root.

The local dynamics in the $(c, L)$ space is illustrated in the phase diagram in Figure 2. From eqs. (10), the $\Delta L_t \equiv L_{t+1} - L_t = 0$ locus has the slope equal to $-w_c(1/\beta - 1)$ and the $\Delta c_t = 0$ line has the slope $-w_c/\beta$. Thus, both are negatively sloped and the former is flatter (as $\omega h' > 1$ in the steady state). Various directions of change in $(c, L)$ shown in Figure 2 are based on eqs. (10). We see that optimal trajectories lie in the regions II and IV. This gives us the following proposition:

\(^{10}\)From (10) we are dealing with the Jacobian matrix

$$D = \begin{pmatrix} 1/\beta & 1/w_c \\ -\omega h''/\sigma & 1 - h''/(\sigma w_a) \end{pmatrix}. $$

By inspection, both Trace $D$ and Det $D$ are positive. Hence both of its roots are positive. Define $E \equiv D - I$. It is easy to derive that Trace $E > 0$ and Det $E < 0$. Hence one root is positive and the other is negative. But the eigenroots of $D$ are one plus the eigenroots of $E$, implying that $D$ has one root in $(0,1)$ and the other exceeding one.
Proposition 1: Along the optimal trajectory, child labor increases and family consumption falls, or child labor decreases and family consumption rises, according as $L_0 \leq L^*$. This tells us that if initially the family is using too little of child labor as compared to the long run equilibrium, its optimal path requires it to increase child labor over time; consequently, successive generations get less and less education. This impoverishes the family and reduces consumption over time. The opposite holds if $L_0 > L^*$.

2.3 Parameter Shifts

Our base-line model will now be used to analyze the following types of policies designed to fight child labor: (a) Lump Sum Subsidies, (b) Adult Wage Increases, (c) Enrollment Subsidies (like the “food for education” programs), (d) Improvements in Primary Education (e.g. expenditures to enhance the education infrastructure at the school level) and (e) A Direct Ban on Child Labor (by legislation and enforcement).

2.3.1 Lump-Sum Subsidy

This is perhaps the simplest anti-poverty measure. The logic underlying this type of policy would be that it would allow a poor family to increase its current consumption, ease its need for consumption smoothing and thus reduce its incentive to send the child to work.

Suppose the government offers the family a direct subsidy $S$ in every time period. The family’s budget constraint changes to $c_t = w_cL_{t+1} + w_a h(1 - L_t) + S$. Notice, however, that this policy fails
to affect the long run marginal condition for intertemporal optimality: eq. (6) still remains the rule describing optimal behavior. Therefore, this policy does not affect child labor in the steady state. The family’s long-run consumption, however, increases.11

Since \( L^* \) remains unchanged, there is a direct jump in consumption from the old steady state level to the new one, i.e., there is no transitional dynamics. Thus, there is no effect of this policy on child labor in the short run either.12

**Proposition 2:** A permanent, lump-sum subsidy to the family does not affect child labor either in the long run or in the short run. The family responds to this program by instantly increasing its consumption by the amount of the subsidy and maintaining that level of consumption.

### 2.3.2 Adult Wage Increase

The existing literature has stressed that policies that result in adult-wage increases are potent in fighting the problem of child labor (Basu and Van (1998)). Adult wage can change through a change in trade policy, for example, or, in the long run, as a consequence of economic development of sectors that lead to increased job opportunities for adults. It can also change, in our model, as a consequence of direct policy interventions like an increase in the minimum wage.

The long-run effects of an increase in the adult wage are easy to see. In Figure 1(a), the \( \omega h' \) curve shifts to the right. Hence child labor falls. It is because an increase in the adult wage raises the return from the child’s education.13 Totally differentiating the budget constraint (7) it can be seen that the long-run family consumption increases.

A decrease in \( L^* \) means that \( L_0 > L^* \). Hence the optimal trajectory lies in region IV of Figure 2. Over time, child labor decreases and family consumption increases.

To determine what happens in the short run, note that family consumption is the “jump” variable. How does it “jump” at \( t = 0 \)? From eq. (11), \( c_0 = c^* - w_c(1/\beta - z_2)(L_0 - L^*) \). Assuming a marginal increase in \( w_a \) (such that \( L^* = L_0 \) initially), we get:

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11Diagramatically, in Figure 1(b) the \( LL \) curve does not shift and the \( CC \) curve shifts to the right.

12This type of policy fails because it cannot to alter the terms governing the long run trade off between current and future consumption. This is partly because \( \beta \) is assumed to be constant. If instead, increased prosperity causes poorer families to attach greater weight to the future (i.e., to increase \( \beta \)), the answer would be different. The case of a variable rate of time preference is considered later.

13As shown by Basu (2000), an increase in the minimum wage may have an adverse impact on child labor in a general equilibrium model allowing for spillovers. Such policies can increase adult unemployment, thus reduce current consumption and thereby induce families to increase child labor to support current consumption levels.
\[ \frac{dc_0}{dw_a} = \frac{dc^*}{dw_a} + wc \left( \frac{1}{\beta} - z_2 \right) \frac{dL^*}{dw_a} \] (12)

Intuitively, on the one hand, as the long-run consumption increases, the consumption-smoothing motive requires \( c_0 \) to increase. On the other, the optimal long-run child labor is at a lower level, and this requires sacrificing consumption in the short run. The algebraic expressions of \( \frac{dc^*}{dw_a} \) and \( \frac{dL^*}{dw_a} \) can be substituted into (12) to give us the following:

\[ \frac{dc_0}{dw_a} = h + (1 - z_2) \frac{h'}{\omega h''} \geq 0. \]

Thus, the net impact on \( c_0 \) is ambiguous and depends on the education technology. In particular, it depends on the reciprocal of the curvature \( \frac{h''}{h'} \) of \( h \). If the marginal returns from education relative to its rate of change are “small”, \( c_0 \) would increase; otherwise it would decrease. There is inadequate empirical evidence to make an assumption one way or another. However, the usual theoretical functional forms for technology, such as \( h = \ln(1 - L) \) or \( (1 - L)^\eta \), \( 0 < \eta < 1 \), together with high levels of child labor would imply that \( c_0 \) would increase. But, regardless of how \( c_0 \) changes, as shown earlier, from \( t = 0 \), \( c_t \) rises monotonically.

2.3.3 Enrollment Subsidy

Under this program, the family obtains a cash or an in-kind subsidy, conditional upon a child’s attendance in school (Ravallion and Wodon (2000)). The food-for-education program in Bangladesh (which is in form of monthly food ration given to the family) and Marenda Escolar, a breakfast-lunch program in schools in Brazil, are examples.\(^14\)

Interpret \( 1 - L \) as attendance in school. Let the total subsidy received by the family, \( S_t \), be an increasing function of \( 1 - L \), i.e. \( S_t = S(1 - L_{t+1}) \), \( S' > 0 \). Following Ravallion and Wodon (2000),\(^15\) for simplicity, let the function \( S \) be proportional: \( S_t = (1 - L_{t+1})\tau, \tau > 0 \).

This kind of program reduces the opportunity cost of educating the child. In the absence of this program, this opportunity cost is the child-wage, \( w_c \). With the program in place it is equal to \( w_c - S' = w_c - \tau \). If \( \tau \geq w_c \), it is zero or negative and thus child labor would stop altogether. However, the cost of such an ambitious program in a developing economy with a large population

\(^{14}\)Enrollment subsidy is also a pilot program being undertaken by World Bank.

\(^{15}\)Note that their model is somewhat different in that it allows for an additional good, leisure.
of poor families would almost certainly be prohibitive. So, as Ravallion and Wodon (2000) do, we too consider the more interesting and realistic case where $\tau < w_c$.

The family’s budget constraint in this case is:

$$c_t \leq w_c L_{t+1} + w_a h(1 - L_t) + (1 - L_{t+1}) \tau.$$  \hfill (13)

The Euler equation governing the dynamics is now given by:

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1 - \tau/w_c}{\omega h'(1-L_{t+1})},$$  \hfill (14)

and accordingly the steady-state level of child labor is determined by:

$$\omega h'(1-L^*) = \frac{1 - \tau/w_c}{\beta}.$$  \hfill (15)

An increase in $\tau$ can be thought of as an introduction or enhancement of this program. We have $dL^*/d\tau < 0$, i.e., a permanent enrollment subsidy program reduces the long-run incidence of child labor – by lowering the opportunity cost of sending the child to school. It is straightforward to show that such a program also increases the long-run level of family consumption.

How does it affect the dynamics of child labor and family consumption? As in case of an increase in the adult wage, $L_0 > L^*$. Thus the optimal trajectory lies in region IV of Figure 2. Accordingly child labor decreases and family consumption increases over time.

The effect on $c_0$ is amenable to a qualitatively similar analysis and interpretation to those of an increase in adult wages. A long-run rise in family consumption tends to increase $c_0$ and long-term decline in child labor tends to lower $c_0$. Notice that in this case we have

$$c_0 = c^* - (w_c - \tau)(1/\beta - z_2)(L_0 - L^*).$$  \hfill (16)

Totally differentiating (16) and (15) and solving, we get:

$$\frac{dc_0}{d\tau} = 1 - L^* + (1 - z_2) \frac{h'}{h''} \geq 0.$$  

**Proposition 3:** The effects of an increase in adult wage and an enrollment subsidy program are qualitatively the same. The long-run incidence of child labor is less and the long-term family consumption is greater. Initially and over time, child labor decreases. Family consumption increases over time, but, in the initial period, it may increase, decrease or remain unchanged. The direction of the change is determined by the curvature ($\frac{h''}{h'}$) of the educational technology function. If the return to education is small relative to its rate of change, the policies have the desirable short term effects both of reducing child labor and of increasing consumption.

\footnote{This is an analog of the expression for $c_t$ in (11) with $t = 0.$}
2.3.4 Improvement in Primary Education

The skill acquired from education depends on educational infrastructure. In developing countries including Brazil, Peru and India there is considerable evidence of the ‘inadequacy’ of infrastructure at the primary and upper primary (middle-school) level – in terms of access to schools, space available for classrooms, teacher resources etc. (see Brown, 2001, The Probe Team, 1999). We now analyze how a policy of enhancing educational infrastructure affects child labor in the long run, short run and over time. Let such a policy be modelled through a multiplicative shift parameter, say, $A$, in the function $h(.)$, i.e. let $h = A\tilde{h}(1 - L_{t+1})$, where $A$ increases from its initial value, say 1.

Now, the steady-state condition determining child labor is:

$$\omega Ah'(1 - L^*) = \frac{1}{\beta}. $$

It is evident that child labor falls in the long run as $A$ increases. Differentiating the family budget constraint it is seen that family consumption in the long run increases too. These are expected outcomes, and, analogous to those of an increase in adult wage or the enrollment subsidy program.

The short-run effects are, however, somewhat different. These differences arise because the benefit from this program to the family in terms of the rise in the adult’s earning starts to accrue from period 1, not instantaneously from period 0 when the policy is first implemented. This has two implications. First, any change in consumption in period 0 must arise entirely from adjustment in child labor in that period. This is because the budget constraint at $t = 0$ remains the same as before the policy is initiated, namely,

$$c_0 = w_c L_1 + w_a h(1 - L_0).$$

(17)

Second, from period 1 onwards the budget constraint is different: $c_t = w_c L_{t+1} + w_a A h(1 - L_t)$, $A > 1$. Hence, from period 1, this equation and the Euler equation (9) govern the optimal trajectory.

The question is, how does the increase in $A$ affect $c_0$ and $L_1$? In principle, eq. (17), together with the Euler equation at $t = 0$ – which takes into account the effect of an increase in $A$ from period 1 onwards – determine these two variables. More specifically, using (11) we get,

$$c_1 = c^* - w_c \left( \frac{1}{\beta} - z_2 \right) (L_1 - L^*) \equiv f(L_1, A).$$

(18)

17Brazil has aggressively pursued a policy of enhancing education infrastructure at the school level (through textbook program like Livro Didactico and teacher-skill raising TV program like Escola). See Brown (2001).
This function has the properties, \( \frac{\partial f}{\partial L_1} = -w_\epsilon(1/\beta - z_2) < 0 \), and for an arbitrarily small increase in \( A \), \( \frac{\partial f}{\partial A} = \partial c^* / \partial A > 0 \), since, at the initial value of \( A \), \( L_1 = L^* \).

Note that from period 1 the parent’s problem is exactly analogous to the original problem, except that the parameter \( A \) has a higher value. Define the value function \( V(L_t; A) \) for \( t \geq 1 \). It has the envelope property

\[
\frac{\partial V}{\partial L_t} = -u'(c_t)w_Ah'(1 - L_t).
\]  

(19)

Then, at \( t = 0 \), the parent’s problem can be stated as \( \max_{c_0, L_1} u(c_0) + \beta V(L_1; A) \), subject to the budget equation (17). The first-order condition of this problem is: \( w_\epsilon u'(c_0) + \beta \frac{\partial V(L_1; .)}{\partial L_1} = 0 \). Using (19) this can be expressed as

\[
u'(c_0) = \beta \omega Ah'(1 - L_1)u'(c_1).
\]  

(20)

We now substitute (18) into (20) and obtain the following Euler equation at \( t = 0 \):

\[
u'(c_0) = \beta \omega Ah'(1 - L_1)u'(f(L_1, A)).
\]  

(21)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Initial Effects of an Improvement in Primary Education}
\end{figure}

Eqs. (17) and (21) determine \( c_0 \) and \( L_1 \). Indeed the former equation generates a positive locus and the latter a negative locus between the two variables. These are depicted in Figure 3 and respectively marked as BB and EE. Now, as \( A \) increases, the BB curve does not shift. Hence the effects on \( c_0 \) and \( L_1 \) depends on how the EE curve shifts. However, from eq. (21) it is not clear which way it shifts. Totally differentiating (17) and (21) gives

\[
\frac{dc_0}{dA} = \frac{\sigma w_\epsilon}{\sigma w_\epsilon(\frac{1}{\beta} + 1 - z_2) - \frac{h'}{h''}} \left[ \omega h - \frac{h'}{h''} \left( \frac{1}{\beta} - 1 \right) \right] - 1 \frac{dL_1}{dA} = \frac{1}{w_c} \frac{dc_0}{dA}.
\]  

(22)
Clearly, the signs are ambiguous. This ambiguity comes about because the long-term increase in consumption and decrease in child labor tend to imply more consumption and less child labor, but at the same time an increase in \( c_0 \) has to be financed from an increase in child labor initially; these two conflicting tendencies result in the ambiguity of the signs of \( dc_0/dA \) and \( dL_1/dA \). Furthermore, note from (22) that, unlike in the case of adult wage increase or enrollment subsidy, the direction of change in \( c_0 \) and \( L_1 \) depends on the curvature of the utility function (\( \sigma \)). Both \( c_0 \) and \( L_1 \) increase or decrease as \( \sigma \) is sufficiently large or small. It is, however, clear that one of the two undesirable short-run effects of the policy – either an increase in child labor or a decrease in consumption – is unavoidable.\(^{18}\)

The dynamics from period 1 depends on whether \( L_1 \gtrless L^* \). Given that \( dL^*/dA < 0 \), if \( dL_1/dA > 0 \), then, unambiguously, \( L_1 > L^* \). If, instead, \( dL_1/dA < 0 \), does it fall below the new long-run level of child labor \( L^* \)? We show next that it does not. Subtracting \( dL^*/dA = h'/h'' \) from the expression of \( dL_1/dA \) in (22),

\[
\frac{dL_1}{dA} - \frac{dL^*}{dA} = \frac{\sigma w_c \left( \omega h - \frac{h'}{h''} \left( \frac{2}{3} - z_2 \right) \right)}{\sigma w_c \left( \frac{1}{3} + 1 - z_2 \right) - \frac{h''}{h'}} > 0.
\]

This implies \( L_1 > L^* \), irrespective of whether child labor initially increases, decreases or remains unchanged. According to Proposition 2 then, from period 1 child labor decreases and consumption increases over time.

One remaining question is: how does consumption change from period 0 to period 1? From (20), \( c_1 \gtrless c_0 \) as \( d[Ah(1 - L_1)]/dA \gtrless 0 \). Using the expression (22) it is easy to verify that this derivative is positive. Hence, \( c_1 > c_0 \).

Figure 4 illustrates the possible patterns of dynamics.

**Proposition 4:** The long-run effects of a program of improving primary education are analogous to those of the enrollment subsidy program or an increase in adult wage: child labor falls and family consumption rises. The impact on initial consumption and dynamics of consumption are also analogous: consumption may increase, decrease or remain unchanged initially but over time it increases. However, with respect to child labor, the initial impact is ambiguous, but from the following period it decreases monotonically over time. Furthermore, initially, almost always, child

---

\(^{18}\)The possibility that both \( c_0 \) and \( L_1 \) do not change is not generic. \( \frac{dc_0}{dA} = \frac{dL_1}{dA} = 0 \) iff the parameters of the model are exactly such that at the initial equilibrium \( \sigma w_c \left[ \omega h - \frac{h'}{h''} \left( \frac{1}{3} - 1 \right) \right] - 1 = 0 \); this will not hold for small perturbations of the parameters of the model and would thus have a probability of zero of being observed in the real world.
labor and family consumption either both increase or they both decrease; the direction of this change is determined by the curvature of the utility function.

Comparing Propositions 3 and 4 we observe that, except for how child labor is affected initially, other effects are qualitatively similar to those of enrollment subsidy or an increase in adult wage. There is, however, an important difference between this program and enrollment subsidy. That is, only a permanent program of enrollment subsidy can reduce child labor in the long run. But a temporary program of enhancing educational infrastructure may have a permanent effect if the improved infrastructure is a stock variable like buildings, labs, classrooms, the curriculum etc. that last over time – rather than a flow like the number of teachers employed.

2.3.5 A Ban on Child Labor

Most countries have some laws against child labor. Even developing countries like India and Brazil have elaborate laws prohibiting child labor. But in these countries, typically, the enforcement is lax and compliance is poor. However, because of pressures from the developed countries and international organizations like the ILO, the developing countries have recently paid more attention to the enforcement problem.

In considering a ban on child labor we will make the following additional – and reasonable – assumption that \( \omega h' > 1 \) for all \( L \in (0, 1) \), i.e., wages and education technology are such that the undiscounted marginal return from child’s education is always positive. This implies that, in Figure

Figure 4: Dynamics arising from Improvement in Primary Education
1(b), the slope of the CC curve, \( dc/dL \) is negative for any \( L \in (0, 1) \).

Suppose now that child labor is partially or wholly banned, i.e., reduced from \( L^* \). Ignoring the incentive problems associated with such a ban, it follows that both in the long run and in the short run child labor falls. Further, from Figure 1(b), we see that in the long-run family consumption increases.

But, note from the family’s budget constraint that its consumption must decline in the short run. Also, the involuntariness of a ban implies that, although child labor is partially or totally eliminated, and the long-run consumption is higher, unlike the other policies discussed so far, this policy unambiguously lowers the family’s welfare in terms of its discounted sum of utilities.

3 Availability of Loans and Child Labor

We have assumed till now that credit market is totally absent. Would child labor disappear – or at least be less – if loans are made available to poor families? It is to this question that we turn next.

Suppose the government introduces a loan program allowing families to borrow at a low interest rate, say \( r \). Assume that at this rate the family can borrow only up to a certain limit, say \( \bar{b} \). Moreover, all loans are one-period loans and there are no informational problems or incentive problems in paying back the loans. The limit \( \bar{b} \) is set low enough so that the family is able to pay any interest that becomes due on the loan.\(^{19}\)

Given access to such loans, the parent’s problem is same as in the base-line model except that the family faces the revised budget constraint given by:

\[
c_t \leq w_c L_{t+1} + w_a h(1 - L_t) + b_{t+1} - (1 + r)b_t, \quad t = 0, 1, \ldots, \infty
\]

and there is a loan constraint:

\[
0 < b_t \leq \bar{b}, \quad t = 1, 2, \ldots, \infty,
\]

where \( b_t \) denotes the principal to be repaid at time \( t \) (i.e., equal to the loan incurred at \( t - 1 \)).

\(^{19}\) A very stringent limit would be to insist that \((1 + r)\bar{b} < w_a h(0)\), that is, an adult, with no education, is able to pay back the interest and the whole principal.
The Lagrangian of the modified problem is given by:

\[
\mathcal{L} = \sum_{t=0}^{\infty} u(c_t)\beta^t + \sum_{t=0}^{\infty} \{ \lambda_t[w_c L_{t+1} + w_a h(1 - L_t) + b_{t+1} - (1 + r)b_t - c_t] \}
\]

\[+ \sum_{t=1}^{\infty} \{ \xi_1 t L_t + \xi_2 t (1 - L_t) + \delta_t (\bar{b} - b_t) \}. \]

Compared to the base-line model, clearly, there is an additional series of choice variables: \( \{b_t\}_1^{\infty} \).

For the variables, \( \{c_t\}_0^{\infty} \) and \( \{L_t\}_1^{\infty} \), we obtain exactly the same first-order conditions (1 to 4) as before. In addition, we get the following two conditions, associated with the choice of the level of debt subject to the borrowing constraint.

\[
\lambda_{t-1} - \delta_t - \lambda_t (1 + r) = 0
\]

\[
\delta_t (\bar{b} - b_t) = 0, \, \delta_t \geq 0.
\]

In the steady state, \( c_t = c^* \), \( L_t = L^* \) and \( b_t = b^* \). As before, eq. (1) reduces to \( \lambda_{t+1}/\lambda_t = \beta \).

Using this relation eq. (25) can be stated as

\[
1 - \beta(1 + r) = \delta_t/\lambda_{t-1}.
\]

The Inada conditions imply \( c_t > 0 \) and thus \( \lambda_{t-1} > 0 \). Hence, if we impose the

**Borrowing Assumption:**

\[
\beta(1 + r) < 1,
\]

it will imply that \( \delta_t > 0 \) and that in the steady state the family will borrow the full amount available to it (i.e., \( \bar{b} = b_t \).\(^{20}\) Given that the borrowing constraint is binding, the family’s budget constraint at \( t = 1, 2, .. \) is given by:

\[
c_t = w_c L_{t+1} + w_a h(1 - L_t) - r\bar{b}.
\]

Since eqs. (1) to (4) are shared with our base-line model and they do not depend on \( r \) and \( \bar{b} \), eq. (6) continues to hold as the long-run rule for child labor. It then follows that the steady-state

\[^{20}\]The interest rate is presumably low and a poor households’ discount factor is likely to be low as well, so that, in the context of our model, it is not unreasonable to require \( \beta(1 + r) < 1 \).
policy conclusions derived in our base-line model remain valid in the presence of limited access to the loan market.\textsuperscript{21}

**Proposition 5:** The extent of child labor in the long run is not affected by the interest rate, $r$, or the borrowing limit, $\bar{b}$.

This proposition, a “neutrality result”, may strike us as surprising. It can, however, be understood as follows. The Borrowing Assumption states that the price for current consumption in terms of future consumption using the available credit, $(1+r)$, is less than the steady-state marginal rate of substitution of current consumption for future consumption $(1/\beta)$. This implies that the family “buys” current consumption using credit up to the limit $\bar{b}$. At this point the constraint becomes binding. The borrowing constraint being binding implies, by itself, that the family has an incentive to increase current consumption further as long as the cost of such an increase is less than the steady state marginal rate of substitution. Hence, increases in current consumption are “financed” using child labor, as long as the returns for education $(\omega h(1-L^*))$ representing (in steady state) the cost of current consumption is less than $1/\beta$. Thus, at the margin, the decision on child labor is dependent only on the marginal rate of substitution between current and future consumption, and the returns in the market for education rather than on the terms governing available loans.

We now analyze the effects of variations in the loan-market parameters.

### 3.1 Decrease in the Interest Rate

Suppose the government lowers $r$ on loans permanently. This doesn’t apply to loans already incurred in period 0, but to subsequent loans. As we have just seen, such a change does not affect the return from education at the margin and hence doesn’t influence the long-run level of child labor. A lower interest rate lowers the cost of debt however. Thus, family consumption increases in the long run.

Since $r_0$ is unchanged, so is the budget constraint facing the family in period 0. Hence the impact of the policy in the short run is somewhat similar to that of the case of an improvement in primary education.

How do $(c_0, L_1)$ change? Since $c^*$ increases, the consumption-smoothing tendency implies a higher $c_0$. In turn, this must be financed by more child labor in period 0 (i.e. child labor increases in the very short run). This implies $L_1 > L^*$.\textsuperscript{22} Therefore, from period 1 onwards child labor

\textsuperscript{21}Indeed, this case is theoretically equivalent to the case of consumption subsidy discussed for our baseline model with the subsidy being viewed as a loan at -100% interest.

\textsuperscript{22}The formal derivation of the short-run effects are given in Appendix A.
decreases and family consumption increases along the optimal trajectory in region IV of Figure 2.

**Proposition 6:** Following a decrease in the interest rate, child labor increases from period 0 to period 1 and then it monotonically falls and converges in the long run to its original level. Family consumption in period 0 jumps up, then it monotonically rises and finally converges in the long run to a higher level compared to the old steady state.

These effects are illustrated in Figure 5.

![Figure 5: Decrease in the Interest Rate](image)

3.2 Increase in the Amount of the Loan

Suppose \( b \) is marginally increased on a permanent basis. That is, at \( t = 0 \) fresh loans available are now equal to \( b + \epsilon \), while the debt already incurred, \( b_0 \), is unchanged. The borrowing constraint being binding, we have \( b_1 = b_2 = ... = b + \epsilon \).

This program, like a change in the interest, also does not influence the return from education and hence does not affect child labor in the long run. But, from the family’s budget constraint we see that the family consumption in the steady state declines – since the total cost of servicing debt is now higher.\(^{23}\) This is exactly the opposite of the corresponding long-term effect of a decrease in the interest rate.

In the short run (period 0), however, the availability of more (fresh) loans expands the scope for family consumption. Hence \( c_0 \) increases. But consumption smoothing over time implies that the increase in \( c_0 \) will be less than the increase in the loans available, \( \epsilon \). As a result, the family reduces child labor initially, i.e. \( L_1 \) declines.\(^{24}\) Hence \( L_1 < L^* \). The dynamics from period 1 onwards follows the saddle-path in region II. Figure 6 illustrates the time paths.

\(^{23}\)There is no possibility of default in the model.

\(^{24}\)These effects are derived in Appendix B.
Proposition 7: Following a marginal increase in the amount of loans available, child labor decreases from period 0 to period 1 and from period 1 it increases monotonically. Moreover, family consumption in period 0 increases and then it decreases monotonically from period 0. Compared to the initial steady state, in the new steady state there is no change in child labor but family consumption is less.

Thus, neither the availability of loans nor marginal changes in the loan market parameters have any impact in our model on the long-run incidence of child labor. But in the short run child labor is affected. Moreover, while a decrease in the borrowing rate serves to reduce child labor in the short run, an increase in the loan limit tends to increase it.

We are not arguing, however, that in a fully dynamic framework with perfect foresight changes in loan market parameters cannot influence child labor in the long run. As we show next they can (as do other policies that we have till now deemed ineffective) if the rate of time preference is variable.\footnote{There is another case where marginal changes in the subsidized loan-market parameters will affect the long-run child labor – namely, when the family is constrained to use child labor in order to pay off a large enough loan incurred in the past, typically from the informal credit market. We do not model or discuss this case.}

4 Variable Rate of Time Preference and Child Labor

A large portion of the literature on child labor recognizes that poor families discount the future much more heavily than richer families, and, it may be argued that policies that alleviate poverty reduce child labor by decreasing the subjective discount rate. While our assumption of a constant discount rate is a fair approximation for examining short run (immediate) impacts of policy changes,
the variability of the discount rate cannot be ignored for long-run policy analysis. We now extend our model to allow for the discount rate to be variable and endogenously determined.

Assume that

\[ \beta_{t+1} = \beta(c_t), \; \beta' > 0 > \beta'', \]

where \([\underline{\beta}, \overline{\beta}], 0 < \underline{\beta} < \overline{\beta} < 1,\) is the range of the function. This function tells us that the higher the current consumption (or utility) enjoyed by the family the less impatient it is or, in other words, the more the family consumes the less it discounts the future. We call this the assumption of decreasing marginal impatience (DMI). It is both intuitively appealing and supported by empirical evidence (e.g., Lawrence (1991), Ogaki-Atkinson (1997) and Samwick (1998)).

Define \(\rho_{0,t} \equiv \beta(c_0)\beta(c_1)\ldots\beta(c_{t-1})\) with the property that \(\rho_{0,0} = 1\). It follows that \(\rho_{t,s} = \rho_{0,s}/\rho_{0,t}\) if \(s > t\). Now define \(\phi_t = \sum_{s=t}^{\infty} \rho_{t,s} u(c_s)\), the discounted sum of utility from period \(t\) onwards. This function has the property that

\[ \phi_t = u(c_t) + \beta(c_t)\phi_{t+1}. \]

We can now state the parent’s problem at \(t = 0\) as Maximize \(\phi_0\), subject to (28), (29) and (30).

Compared to our basic model where the discount factor is fixed, there is an additional set of choice variables, \(\{\phi_t\}_{0}^{\infty}\). Setting up the Lagrangian and proceeding as before gives us the following Euler equation:

\[ \frac{\beta(c_t)[u'(c_{t+1}) + \beta'(c_{t+1})\phi_{t+2}]}{u'(c_t) + \beta'(c_t)\phi_{t+1}} = \frac{1}{\omega h'(1 - L_{t+1})}. \]  

If the borrowing constraint is binding, then Eqs. (28), (30) and (31) govern the dynamics of the system. They contain three variables, \(c_t, L_t\) and \(\phi_t\). The steady state is defined by

\[ c^* = w_c L^* + w_a h(1 - L^*) - r \overline{v}, \]  

26 Until recently the general practice has been to assume increasing marginal impatience, if the discount factor is variable at all, e.g., Uzawa (1968), Lucas and Stockey (1984) and Obstfeld (1990). Although this assumption is counterintuitive, it has been justified as being “needed” to ensure stability of the steady state. However, recent research by Chakrabarty (2002) and Das (2003) shows that DMI can be consistent with stability. This is also borne out by the model in this paper. In solving the dynamic optimization problem, we follow Chakrabarty (2002).
\[ \omega h'(1 - L^*) = \frac{1}{\beta(c^*)}, \tag{33} \]

where eq. (33) is obtained from (31).

From (32) and (33), we can see that the variable rate of time preference has created an additional channel through which policies may impact child labor in the long run – through their impact on \(c^*\) and thereby on \(\beta\). Policy initiatives that we have so far judged as being ineffective can reduce child labor. Of course, policies that were effective with a constant discount rate become even more so. An implication of this is that if we compare any two policies, one from the formerly effective and the other from the formerly ineffective category, and if both policy changes are such that they result in the same change in the long run consumption, then the formerly effective policy results in a larger reduction in child labor.

To see how this works, let us compare an increase in \(w_a\) (“effective” policy) to a reduction in \(r\) (“ineffective” policy), both of which will reduce \(L^*\) and increase \(c^*\). Totally differentiating (33) with respect to \(w_a\) and \((-r)\), we get

\[
\frac{dL^*}{dw_a} = \frac{h'}{w_a h''} + \frac{h'}{\beta h''} \beta'(c^*) \frac{dc^*}{dw_a}; \quad \frac{dL^*}{d(-r)} = \frac{h'}{\beta h''} \beta'(c^*) \frac{dc^*}{d(-r)}.
\]

Thus, if the magnitudes of changes in \(w_a\) and \(r\) are such that the change in \(c^*\) is the same, then \(|dL^*/dw_a| - |dL^*/d(-r)| = -h'/(w_a h'') > 0\) represents the greater impact of an increase in \(w_a\) in reducing child labor.

The above argument presumes that the steady states are stable. To investigate stability, note that eq.(32) gives us a schedule similar to the \(CC\) schedule, which is negatively sloped (locally). Eq. (33) yields the counterpart of the vertical \(LL\) schedule in Figure 1. DMI implies that this curve is downward sloping. It is denoted as \(\tilde{L}\tilde{L}\) in Figure 7.

Both schedules being downward sloping the steady state may or may not be unique. Around any steady state however, the dynamics is unstable or saddle-path stable according as the \(\tilde{L}\tilde{L}\) is steeper or flatter than the \(CC\) schedule. This is proved in Appendix C. Figure 7 demonstrates the case of a unique steady state and also a case of multiple steady states. In panel (b) we see that the steady states \(S_1\) and \(S_2\) are stable, while \(U\) is unstable.\(^{27}\)

\(^{27}\)Using panel (b) of Figure 7, we can now formally state the condition for the borrowing constraint to be binding. Among the steady states, consider the ones that are stable. Of these select the one that entails lowest child labor and highest consumption (such as the point \(S_1\)). Let \(c^{**}\) be the associated level of consumption. The Borrowing Assumption is that \(\beta(c^{**})(1 + r) < 1\).
Focussing on a single stable equilibrium, one can see that different ways of relaxing the credit constraint can have opposite effects on child labor and family consumption. Consider first a marginal decrease in the interest rate. In the neighborhood of a stable steady state, it shifts the $CC$ curve to the right. As a result, $c^*$ increases and $L^*$ declines. Intuitively a decrease in the interest rate raises the long-run family consumption and thereby lower the family’s subjective discount rate. This increases the value attached to the return from the child’s education in terms of current consumption, and encourage the family to reduce child labor.  

An increase in the loan-limit does the opposite (as it shifts the $CC$ curve to the left). Not only does the long-run family consumption fall, as the family discounts the future more, it increases child labor.

**Proposition 8:** Under decreasing marginal impatience, a marginal decrease in the interest rate reduces child labor and increases consumption in the long run. An increase in the loan-limit does exactly the opposite.

In other words, compared to the neutrality result under constant rate of time preference, under DMI, loan market intervention of one kind reduces child labor, whereas that of another kind worsens the problem.

The case of multiple stable equilibria gives rise to the possibility of a poverty trap. This does not arise from interactions in the labor markets (as in Basu and Van (1998)), fertility choices (as in Dessy (2000) and Hazan and Berdugo (2002)) or imperfections in the credit market (as in Galor and Zaira (1993)). Decreasing marginal impatience is the underlying reason.  

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28 A lump-sum subsidy works the same way.

29 Recall that our notion of equilibrium is that of a dynamic steady state (like in Galor and Zaira) rather than that.
of consumption, the family discounts the future heavily. This lowers its return from the child’s education and encourages child labor. In turn, this perpetuates poverty and child labor.

In the presence of a poverty trap, we can distinguish between two ways in which child labor can be reduced. Firstly, permanent policy changes that shift the \( \tilde{L} \) curve to the left can completely eliminate the poverty trap. As a result, a marginal policy change may lead to a discrete reduction in the long run level of child labor. Secondly, a temporary policy shock (which does not alter the long run values of the steady states) may nevertheless have a permanent effect – moving the family from a low-consumption and high-child-labor equilibrium to a high-consumption and low-child-labor equilibrium.

5 Summary and Concluding Remarks

In relation to the existing theoretical literature on child labor the distinguishing features of this paper are its infinite horizon framework and the array of policies considered and compared. While laws and mandates requiring a partial or a total ban of child labor “solves” the problem, such a solution imposes short run hardship on families least able to afford it – by reducing their consumption. In the absence of variable discount rates such policies also unambiguously reduce welfare of the families and given their involuntary nature will generate incentive effects that will make them costly to implement. In our view, contrary to that prevailing in some of the literature, from both a normative and a positive perspective such bans are best eschewed because better, non-coercive, alternatives are available. These alternatives are, from the point of view of the affected families, unambiguously welfare-improving.

Our dynamic analysis offers a general insight that non-coercive policies can permanently reduce child labor by operating through two different channels. They may work by changing long run intertemporal terms of trade between current consumption and future consumption. They also work indirectly by altering the family’s time preference in favor of future consumption.

Table 1 represents the first effect. It presents a summary of long-term and short-term effects under the assumption of constant rate of time preference. If the main aim of the policy intervention is to reduce child labor in the long run, we see that a general anti-poverty measure like a lump-sum income subsidy to a poor family, a marginal decrease in the borrowing rate or an increase in the borrowing limit have no impact. On the other hand, adult wage increases, enrollment subsidies

of a Nash equilibrium.
and improvements in the quality of primary education do reduce child labor in the long run.

Political success of particular policies as well as their short run normative justification depend on their short run impacts both on child labor and on the consumption levels of the poor households employing child labor. On this ground, interestingly, improvement in primary education, on its own, does not rate favorably: Either an adverse effect on family consumption or on child labor is unavoidable in the short run.\textsuperscript{30} This leaves enrollment subsidy and policies that increase adult wage as two policies which unambiguously reduce child labor both in the long run and in the short run and which can under some circumstances also increase short run consumption.

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<td>( L: \downarrow ); ( C: \uparrow )</td>
<td>( L: \downarrow ); ( C: \uparrow )</td>
<td>((L: \uparrow, C: \uparrow)) or ((L: \downarrow, C: \downarrow))</td>
<td>( L: \uparrow; C: \uparrow )</td>
<td>( L: \downarrow; C: \uparrow )</td>
</tr>
<tr>
<td>Long Run</td>
<td>( L: \downarrow; C: \uparrow )</td>
<td>( L: \downarrow; C: \uparrow )</td>
<td>( L: \downarrow; C: \uparrow )</td>
<td>( L: \downarrow; C: \uparrow )</td>
<td>( L: \downarrow; C: \uparrow )</td>
<td>( L: \uparrow; C: \uparrow )</td>
</tr>
</tbody>
</table>

Allowing for decreasing marginal impatience (DMI) has three general implications. First, policies that are neutral in case of constant discount rate can have an impact on child labor and policies that reduce child labor when the rate of time preference is constant, are, in the presence of DMI, even more effective. Second, multiple steady states with a poverty-trap situation may arise. This implies that temporary measures may have permanent effects. Third, despite the emphasis in the existing literature on the role of capital market distortions in creating child labor, our analysis indicates that capital market interventions need to be carefully implemented (if at all) and that it may be more fruitful to pursue more direct policies such as ones that subsidize enrollment and/or improve adult wages.

\textsuperscript{30}This is similar to Jafarey and Lahiri (2000), who, based on a model driven by credit constraints, propose that greater emphasis needs to be placed on food for education, relative to investment in primary education. But their conclusion is based on completely different arguments.
Appendices

A  Short-Run Effects of a Decrease in the Interest Rate

Let the interest rate change from $r$ to $r - \gamma$. An increase in $\gamma$ captures a decrease in the interest rate. Proceeding along the same lines as case of improvement in primary education, we have the following two equations determining $c_0$ and $L_1$.

\[
c_0 = w_c L_1 + w_o h(1 - L_0) - r \bar{e}; \quad u'(c_0) = \beta \omega h'(1 - L_1)u'(f(L_1, \gamma)).
\]

Recall that $f(.) = c_1$. We have $\partial f / \partial \gamma = \partial c^* / \partial \gamma = \bar{b} > 0$. Thus, only the EE curve in Figure 3 shifts and it shifts to the right. This implies $dc_0/d\gamma > 0$ and $dL_1/d\gamma > 0$.

B  Short Run Effects of Increase in the Loan-Limit

We proceed as in Appendix A. The equations determining $c_0$ and $L_1$ are the period 0 budget constraint and the Euler equation:

\[
c_0 = w_c L_1 + w_o h(1 - L_0) + \bar{e} - (1 + r)\bar{e}; \quad u'(c_0) = \beta \omega h'(1 - L_1)u'(f(L_1, \bar{b} + \epsilon)). \tag{A1}
\]

\[
u'(c_0) = \beta \omega h'(1 - L_1)u'(f(L_1, \bar{b} + \epsilon)). \tag{A2}
\]

As in Appendix A, $f(.)$ here is consumption in period 1, and, it has the property, $\partial f / \partial \epsilon = \partial c^* / \partial \epsilon = -r < 0$. We see that the graphs (BB and EE) representing the two equations shift as a result of an increase in $\epsilon$, both shifting to the left. Clearly, $L_1$ falls and thus $L_1 < L^*$. The impact on $c_0$ is not clear graphically. However, by totally differentiating these equations:

\[
\frac{dc_0}{d\epsilon} = \frac{1/\beta - z_2 - r - h''/(\sigma h')}{{w}_c(1/\beta + 1 - z_2) - h''/(\sigma h')}
\]

The denominator is clearly positive. Consider the numerator. By our Borrowing assumption, $1/\beta > 1 + r$. Thus $1/\beta - z_2 - r > 1 - z_2$. But $z_2 < 1$. Hence $1 - z_2 > 0$, implying $1/\beta - z_2 - r > 0$ and $dc_0/d\epsilon$ positive.
C Variable Time Preference Model

We will prove that, under decreasing marginal impatience, the steady state is unstable or saddle-path stable as the $\tilde{L}$ curve is flatter or steeper than the $CC$ curve.

The absolute values of the slopes of these curves (at the steady state) are:

$$\left|\frac{dc}{dL}\right|_{LL} = -\frac{\beta h''}{\beta' h'}; \quad \text{and} \quad \left|\frac{dc}{dL}\right|_{CC} = w_c(\omega h' - 1) = w_c \left(\frac{1}{\beta} - 1\right),$$

implying

$$\left|\frac{dc}{dL}\right|_{LL} \lesssim \left|\frac{dc}{dL}\right|_{CC} \quad \text{as} \quad \frac{h''}{w_c h'} + \left(\frac{1}{\beta} - 1\right) \frac{\beta'}{\beta} \equiv \gamma \gtrless 0.$$

We then need to show that all roots of the dynamic system below have modulus greater than one or exactly one root has modulus less than one according as $\gamma \gtrless 0$.

Define $\varphi_t = \varphi_{t+1}$. Substituting this into (30) and (31) we obtain

\begin{align*}
\varphi_t &= u(c_{t+1}) + \beta(c_{t+1})\varphi_{t+1} \\
\frac{\beta(c_t)[u'(c_{t+1}) + \beta'(c_{t+1})\varphi_{t+1}]}{u'(c_t) + \beta'(c_t)\varphi_t} &= \frac{1}{\omega h'(1 - L_{t+1})}.
\end{align*}

These two equations, together with the budget constraint (28) constitute a $3 \times 3$ first-order difference equation system. Totally differentiating three equations and solving, defining $k \equiv -(u'' + \varphi'') > 0$ and $J \equiv \frac{\beta k}{u' + \varphi'} + \beta' > 0$, we get

\begin{align*}
\begin{pmatrix}
dL_{t+1} \\
dc_{t+1} \\
d\varphi_{t+1}
\end{pmatrix}
= A
\begin{pmatrix}
dL_t \\
dc_t \\
d\varphi_t
\end{pmatrix},
\end{align*}

where

$$A \equiv \begin{pmatrix}
\frac{1}{\beta} & \frac{1}{w_c} & 0 \\
\frac{-h''}{\beta h'} & 1 - \frac{\beta h''}{Jw_c h'} & \frac{(1-\beta)\beta'}{J(u' + \varphi')} \\
\frac{(u' + \varphi')h''}{Jw_c h'} & -(u' + \varphi')\left(\frac{1}{\beta} - \frac{h''}{Jw_c h'}\right) & 1 + \frac{(1-\beta)k}{J(u' + \varphi')} \\
\end{pmatrix}.$$

Let $\theta$ denote the eigenroot of $A$. On simplifying, the characteristic equation $|A - \theta I| = 0$ can be expressed as

$$\left(\frac{1}{\beta} - \theta\right) \left\{ \theta^2 - \left[2 + \frac{(1-\beta)k}{J(u' + \varphi')} - \frac{\beta h''}{Jw_c h'}\right] \theta + \frac{1}{\beta} \right\} = 0.$$

Thus one of the roots, say $\theta_3$, is equal to $1/\beta$, which exceeds one. The other two roots are the solutions to the quadratic equation

$$Q(\theta) \equiv \theta^2 - \left[2 + \frac{(1-\beta)k}{J(u' + \varphi')} - \frac{\beta h''}{Jw_c h'}\right] \theta + \frac{1}{\beta} = 0.$$
Let \( \theta_1 \) and \( \theta_2 \) denote its roots. We see that \( Q(0) > 0, Q'(0) < 0 \) and

\[
Q(1) = \frac{\beta \gamma}{J},
\]

where recall that \( \gamma = \frac{h''}{w c h'} + \left( \frac{1}{J} - 1 \right) \frac{\beta'}{\beta}. \) If \( \gamma < 0, \) then \( Q(1) < 0, \) implying that \( \theta_1 \) and \( \theta_2 \) are real, and, one of them exceeds one and the other is less than one. This case is exhibited in Figure 8(a). Given \( \theta_3 > 1, \) it follows that exactly one of the three roots has modulus less than one and hence the steady state is saddle-path stable.

\[
\begin{align*}
\gamma < 0; & \ 0 < \theta_1 < 1 < \theta_2 \\
\gamma > 0; & \ \theta_1, \theta_2 > 1 \\
\gamma > 0; \text{complex roots;} & \ \text{modulus} > 1
\end{align*}
\]

Figure 8: Eigenroots in the Variable Rate of Time Preference Model

If \( \gamma > 0, \) \( \theta_1 \) and \( \theta_2 \) are either real or complex conjugates. These are illustrated respectively in panel (b) and panel (c) of Figure 8. If real, both exceed one. If complex, their modulus, equals \( \frac{1}{\beta} \) (from eq. (A6)). In either case, all three roots have modulus greater than one, implying that the steady state is unstable.

In summary, the steady state is unstable or saddle-path stable as \( \gamma \gtrless 0. \) Earlier, it was established that the \( \tilde{L}L \) curve is flatter or steeper than the \( CC \) curve as \( \gamma \gtrless 0. \) Hence the steady state is unstable or saddle-path stable according as the \( \tilde{L}L \) curve is flatter or steeper than the \( CC \) curve. ■
References


